Chapter 6
Applications of Integration
6.2 Determining Volumes by Slicing

Section Exercises

58. Derive the formula for the volume of a sphere using the slicing method.
Answer: This is a proof; therefore, no answer is provided.

59. Use the slicing method to derive the formula for the volume of a cone.
Answer: This is a proof; therefore, no answer is provided.

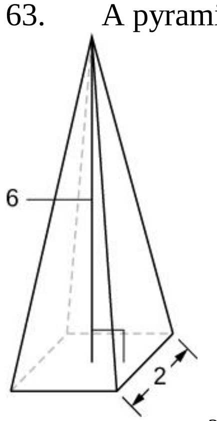
60. Use the slicing method to derive the formula for the volume of a tetrahedron with side length a .
Answer: This is a proof; therefore, no answer is provided.

61. Use the disk method to derive the formula for the volume of a trapezoidal cylinder.
Answer: This is a proof; therefore, no answer is provided.

62. Explain when you would use the disk method versus the washer method. When are they interchangeable?
Answer: This is a proof; therefore, no answer is provided.

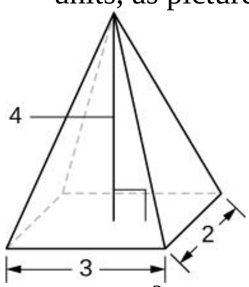
For the following exercises, draw a typical slice and find the volume using the slicing method for the given volume.

63. A pyramid with height 6 units and square base of side 2 units, as pictured here.



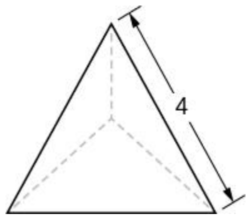
Answer: 8 units^3

64. A pyramid with height 4 units and a rectangular base with length 2 units and width 3 units, as pictured here.



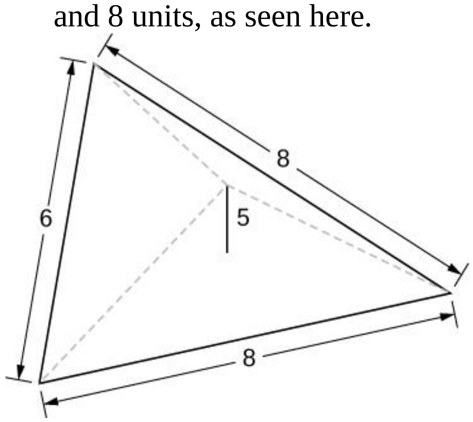
Answer: 8 units^3

65. A tetrahedron with a base side of 4 units, as seen here.



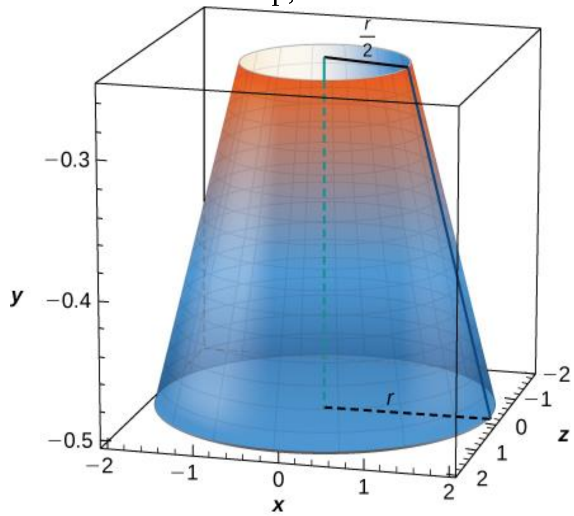
Answer: $\dfrac{32}{3\sqrt{2}}$ units^3

66. A pyramid with height 5 units, and an isosceles triangular base with lengths of 6 units and 8 units, as seen here.



Answer: $5\sqrt{55}$ units^3

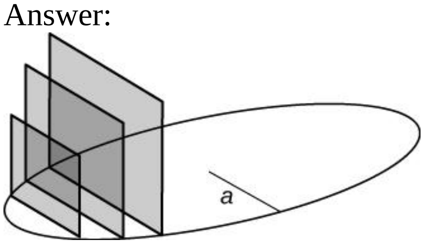
67. A cone of radius r and height h has a smaller cone of radius $r/2$ and height $h/2$ removed from the top, as seen here. The resulting solid is called a *frustum*.



Answer: $\frac{7\pi}{12}hr^2$ units^3

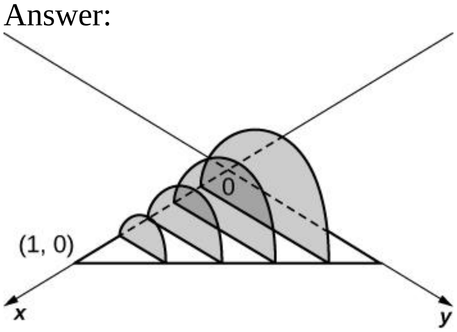
For the following exercises, draw an outline of the solid and find the volume using the slicing method.

68. The base is a circle of radius a. The slices perpendicular to the base are squares.
Answer:



$\frac{16}{3}a^3$ units^3

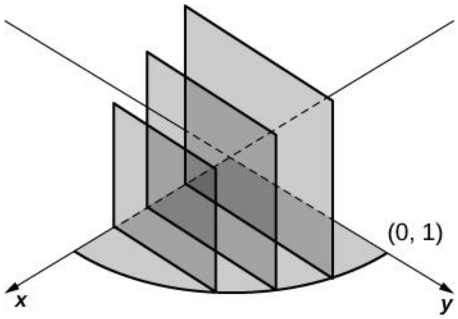
69. The base is a triangle with vertices $(0,\ 0)$, $(1,\ 0)$, and $(0,\ 1)$. Slices perpendicular to the *x*-axis are semicircles.
Answer:



$\frac{\pi}{24}$ units^3

70. The base is the region under the parabola $y = 1 - x^2$ in the first quadrant. Slices perpendicular to the *x*-axis are squares.

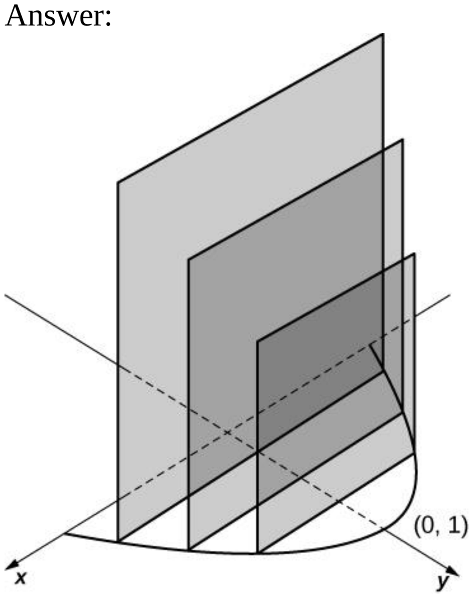
Answer:



$\frac{8}{15}$ units^3

71. The base is the region under the parabola $y = 1 - x^2$ and above the x-axis. Slices perpendicular to the y-axis are squares.

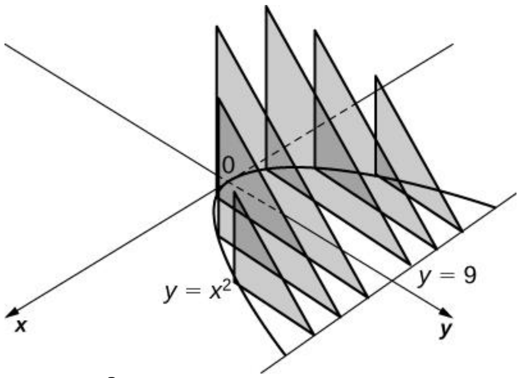
Answer:



2 units^3

72. The base is the region enclosed by $y = x^2$ and $y = 9$. Slices perpendicular to the *x*-axis are right isosceles triangles. The intersection of one of these slices and the base is the leg of the triangle.

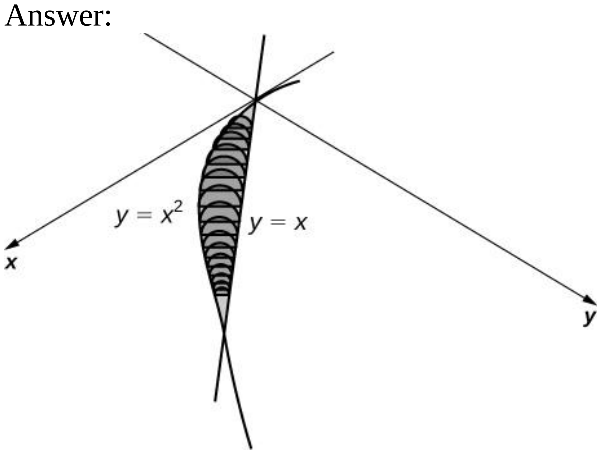
Answer:



9 units^3

73. The base is the area between $y = x$ and $y = x^2$. Slices perpendicular to the *x*-axis are semicircles.

Answer:

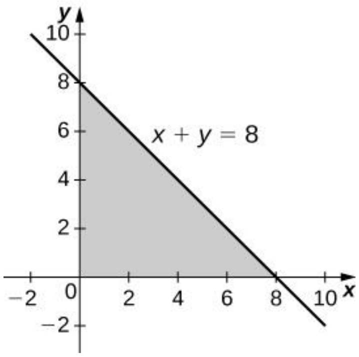


$\frac{\pi}{240}$ units^3

For the following exercises, draw the region bounded by the curves. Then, use the disk method to find the volume when the region is rotated around the *x*-axis.

74. $x+y=8,\ x=0,$ and $y=0$

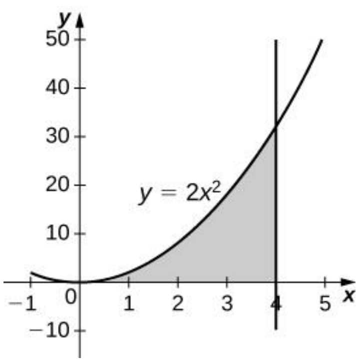
Answer:



$\frac{512\pi}{3}$ units^3

75. $y=2x^2,\ x=0,\ x=4,$ and $y=0$

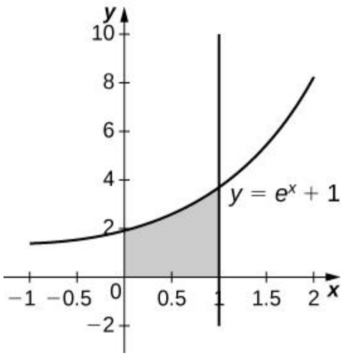
Answer:



$\frac{4096\pi}{5}$ units^3

76. $y = e^x + 1,\ x = 0,\ x = 1,\ \text{and } y = 0$

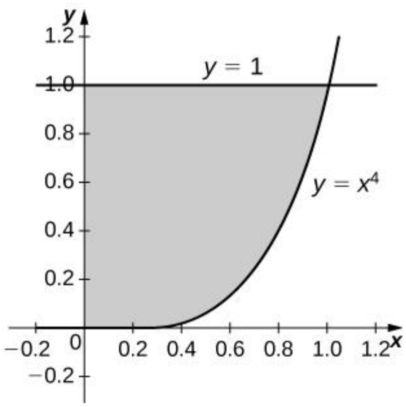
Answer:



$\frac{\pi}{2}\left(e^2 + 4e - 3\right)$ units^3

77. $y = x^4,\ x = 0,\ \text{and } y = 1$

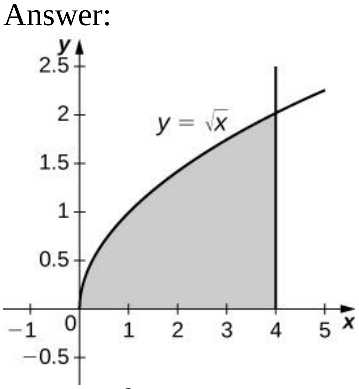
Answer:



$\frac{8\pi}{9}$ units^3

78. $y = \sqrt{x},\ x = 0,\ x = 4,\ \text{and } y = 0$

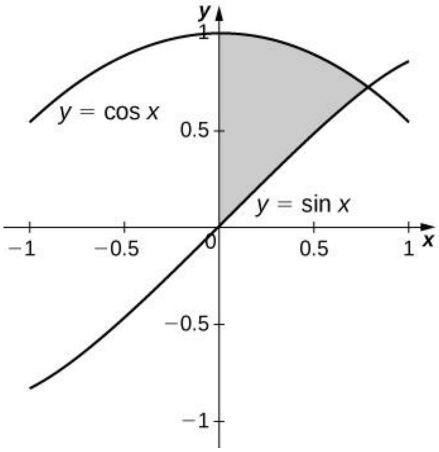
Answer:



8π units^3

79. $y = \sin x,\ y = \cos x,\ \text{and } x = 0$

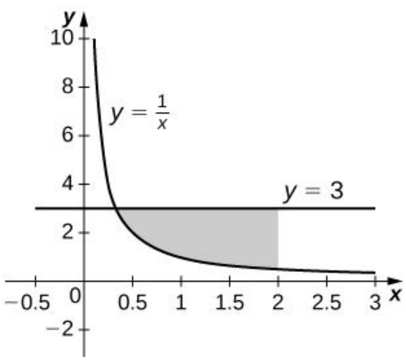
Answer:



$\dfrac{\pi}{2}$ units^3

80. $y = \dfrac{1}{x},\ x = 2,\ \text{and } y = 3$

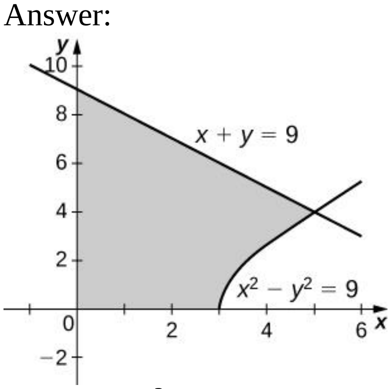
Answer:



$\dfrac{25\pi}{2}$ units^3

81. $x^2 - y^2 = 9$ and $x + y = 9, y = 0$ and $x = 0$

Answer:

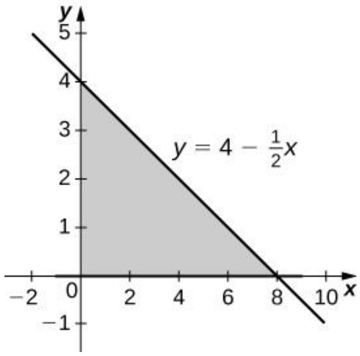


207π units^3

For the following exercises, draw the region bounded by the curves. Then, find the volume when the region is rotated around the *y*-axis.

82. $y = 4 - \frac{1}{2}x,\ x = 0,$ and $y = 0$

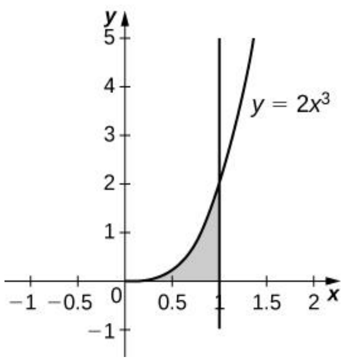
Answer:



$\frac{256\pi}{3}$ units^3

83. $y = 2x^3,\ x = 0,\ x = 1,$ and $y = 0$

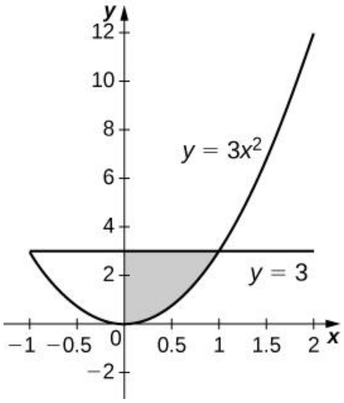
Answer:



$\frac{4\pi}{5}$ units^3

84. $y = 3x^2,\ x = 0,\ \text{and } y = 3$

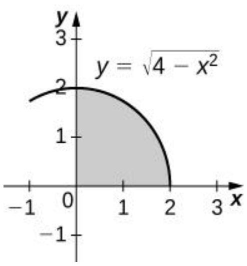
Answer:



$\frac{3\pi}{2}$ units^3

85. $y = \sqrt{4 - x^2},\ y = 0,\ \text{and } x = 0$

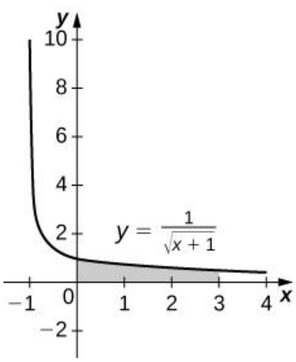
Answer:



$\frac{16\pi}{3}$ units^3

86. $y = \frac{1}{\sqrt{x+1}},\ x = 0,\ \text{and } x = 3$

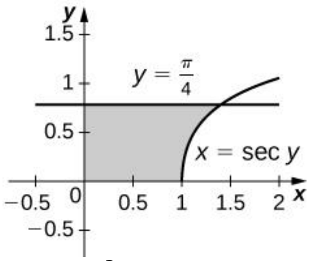
Answer:



$\frac{16\pi}{3}$ units3

87. $x = \sec(y)$ and $y = \dfrac{\pi}{4}$, $y = 0$ and $x = 0$

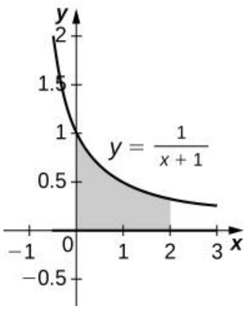
Answer:



π units^3

88. $y = \dfrac{1}{x+1}$, $x = 0$, and $x = 2$

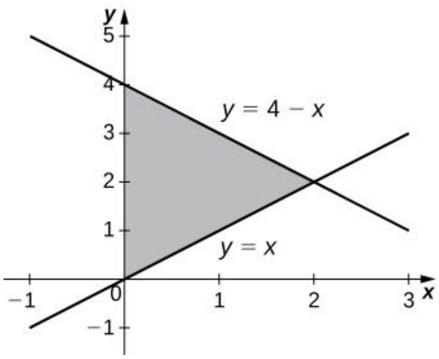
Answer:



$2\pi\left(2 - \ln(3)\right)$ units^3

89. $y = 4 - x$, $y = x$, and $x = 0$

Answer:

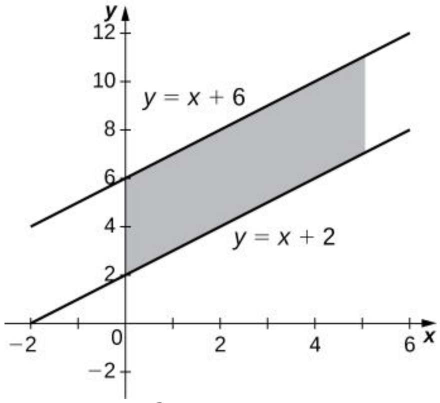


$\dfrac{16\pi}{3}$ units^3

For the following exercises, draw the region bounded by the curves. Then, find the volume when the region is rotated around the *x*-axis.

90. $y = x + 2,\ y = x + 6,\ x = 0,$ and $x = 5$

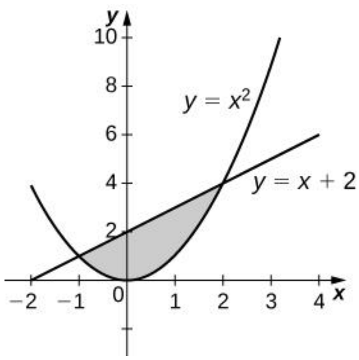
Answer:



260π units^3

91. $y = x^2$ and $y = x + 2$

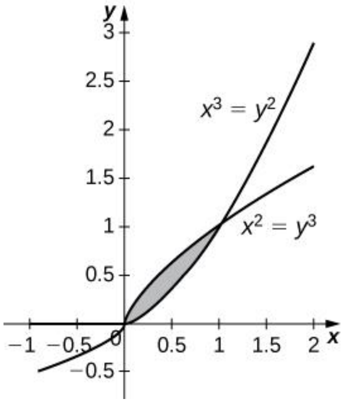
Answer:



$\frac{72\pi}{5}$ units^3

92. $x^2 = y^3$ and $x^3 = y^2$

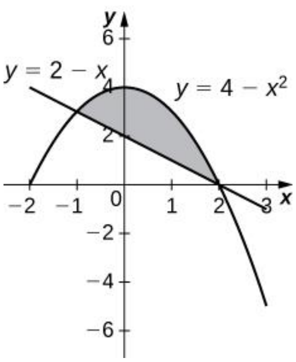
Answer:



$\frac{5\pi}{28}$ units^3

93. $y = 4 - x^2$ and $y = 2 - x$

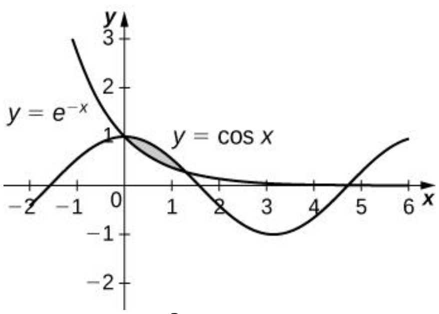
Answer:



$\frac{108\pi}{5}$ units^3

94. **[T]** $y = \cos x,\ y = e^{-x},\ x = 0,$ and $x = 1.2927$

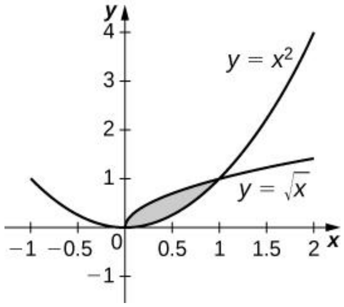
Answer:



0.9928 units^3

95. $y=\sqrt{x}$ and $y=x^2$

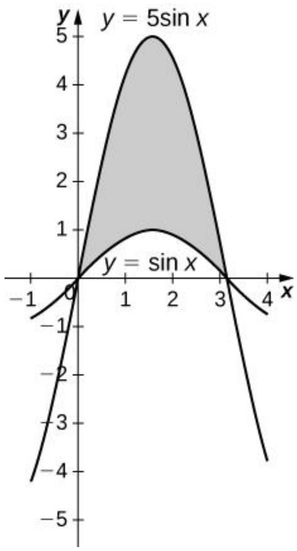
Answer:



$\frac{3\pi}{10}$ units^3

96. $y=\sin x$, $y=5\sin x$, $x=0$ and $x=\pi$

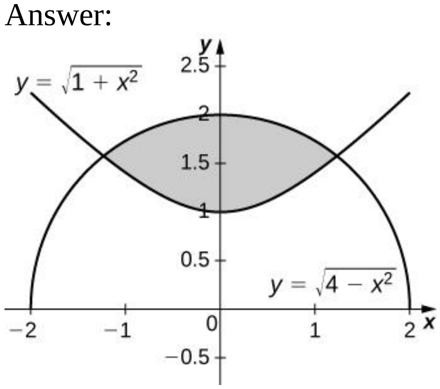
Answer:



$12\pi^2$ units^3

97. $y=\sqrt{1+x^2}$ and $y=\sqrt{4-x^2}$

Answer:

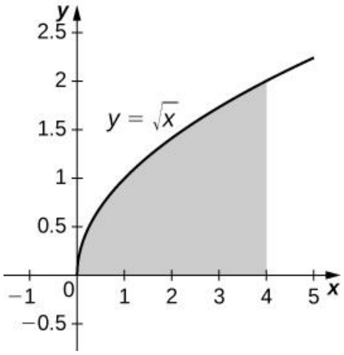


$2\sqrt{6}\pi$ units^3

For the following exercises, draw the region bounded by the curves. Then, use the washer method to find the volume when the region is revolved around the *y*-axis.

98. $y = \sqrt{x},\ x = 4,\ \text{and } y = 0$

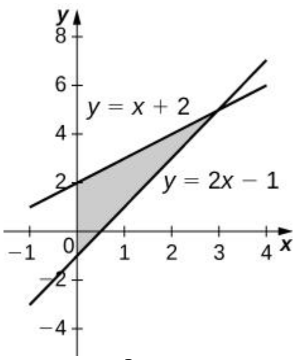
Answer:



$\frac{128\pi}{5}$ units^3

99. $y = x + 2,\ y = 2x - 1,\ \text{and } x = 0$

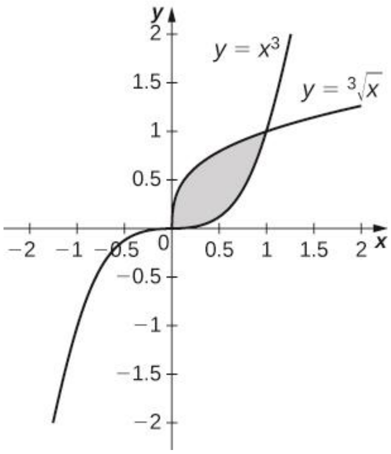
Answer:



9π units^3

100. $y = \sqrt[3]{x}$ and $y = x^3$

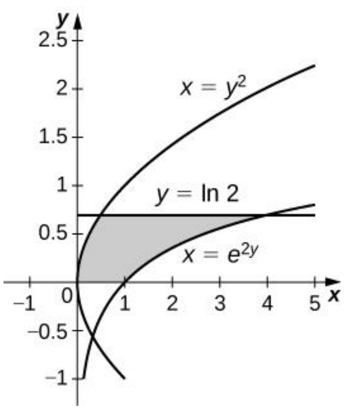
Answer:



$\dfrac{16\pi}{35}$ units^3

101. $x = e^{2y}$, $x = y^2$, $y = 0$, and $y = \ln(2)$

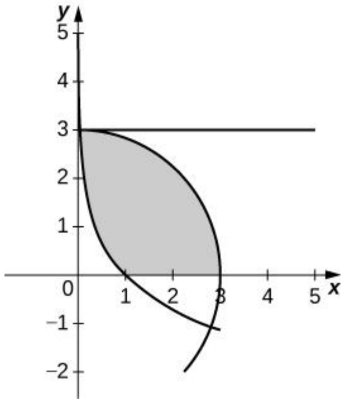
Answer:



$\dfrac{\pi}{20}\left(75 - 4\ln^5(2)\right)$ units^3

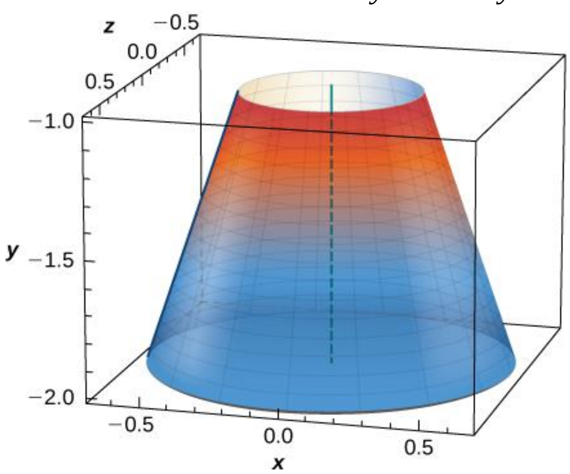
102. $x=\sqrt{9-y^2}$, $x=e^{-y}$, $y=0$, and $y=3$

Answer:



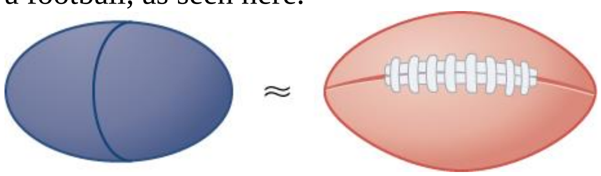
$\frac{\pi}{2e^6}\left(35e^6+1\right)$ units^3

103. Yogurt containers can be shaped like frustums. Rotate the line $y=\frac{1}{m}x$ around the y-axis to find the volume between $y=a$ and $y=b$.



Answer: $\frac{m^2\pi}{3}\left(b^3-a^3\right)$ units^3

104. Rotate the ellipse $\left(x^2/a^2\right)+\left(y^2/b^2\right)=1$ around the *x*-axis to approximate the volume of a football, as seen here.

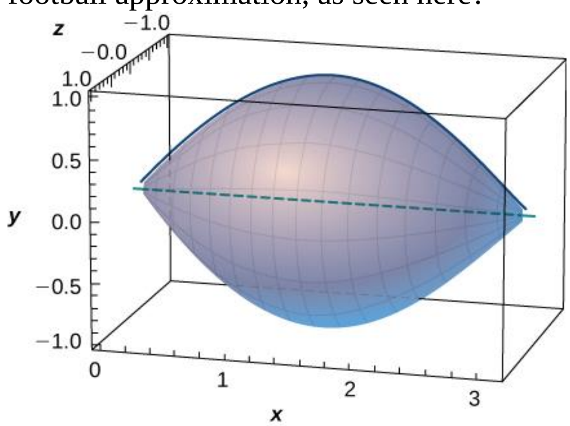


Answer: $\frac{4ab^2\pi}{3}$ units^3

105. Rotate the ellipse $\left(x^2/a^2\right)+\left(y^2/b^2\right)=1$ around the *y*-axis to approximate the volume of a football.

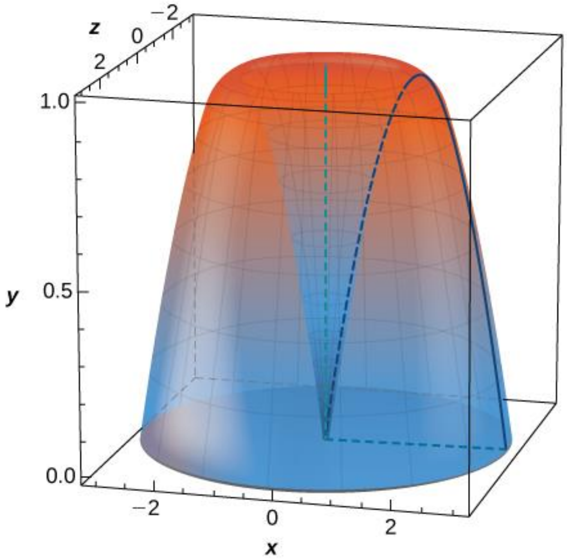
Answer: $\frac{4a^2b\pi}{3}$ units^3

106. A better approximation of the volume of a football is given by the solid that comes from rotating $y=\sin x$ around the *x*-axis from $x=0$ to $x=\pi$. What is the volume of this football approximation, as seen here?



Answer: $\frac{\pi^2}{2}$ units^3

107. What is the volume of the Bundt cake that comes from rotating $y = \sin x$ around the *y*-axis from $x = 0$ to $x = \pi$?



Answer: $2\pi^2$ units^3

For the following exercises, find the volume of the solid described.

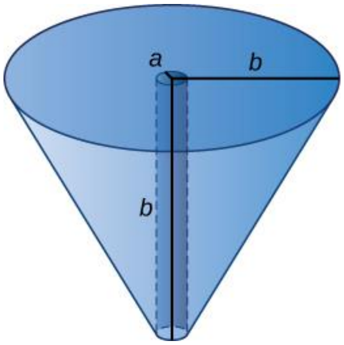
108. The base is the region between $y = x$ and $y = x^2$. Slices perpendicular to the *x*-axis are semicircles.

Answer: $\frac{\pi}{120}$ units^3

109. The base is the region enclosed by the generic ellipse $\left(x^2 / a^2\right) + \left(y^2 / b^2\right) = 1$. Slices perpendicular to the *x*-axis are semicircles.

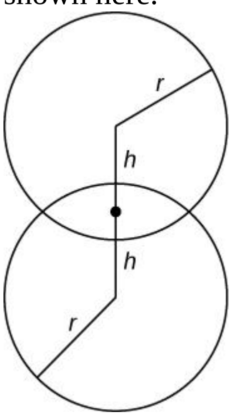
Answer: $\frac{2ab^2\pi}{3}$ units^3

110. Bore a hole of radius a down the axis of a right cone and through the base of radius b, as seen here.



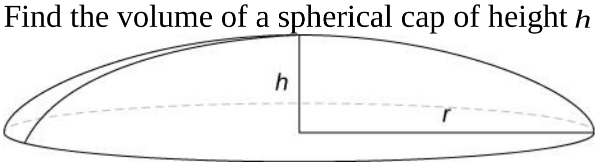
Answer: $\frac{\pi b}{3}(b^2 - 3a^2)$ units^3

111. Find the volume common to two spheres of radius r with centers that are $2h$ apart, as shown here.



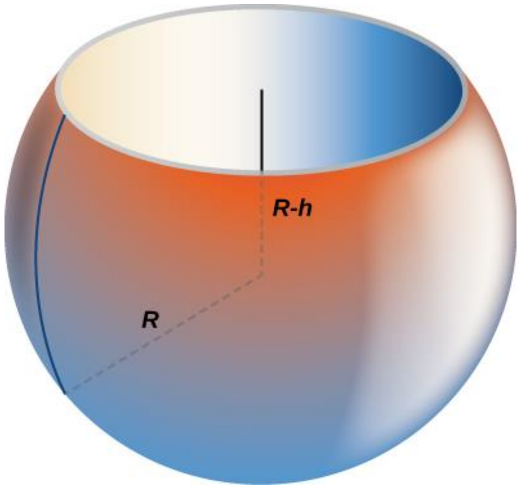
Answer: $\frac{\pi}{12}(r+h)^2(6r-h)$ units^3

112. Find the volume of a spherical cap of height h and radius r where $h < r$, as seen here.



Answer: $\frac{\pi h^2}{3}(3r-h)$

113. Find the volume of a sphere of radius R with a cap of height h removed from the top, as seen here.



Answer: $\frac{\pi}{3}(h+R)(h-2R)^2$ units^3