Chapter 6
Applications of Integration
6.5 Physical Applications

Section Exercises
For the following exercises, find the work done.

218. Find the work done when a constant force $F = 12$ lb moves a chair from $x = 0.9$ to $x = 1.1$ ft

Answer: 2.4 ft-lb

219. How much work is done when a person lifts a 50 lb box of comics onto a truck that is 3 ft off the ground?

Answer:150 ft-lb

220. What is the work done lifting a 20 kg child from the floor to a height of 2 m? (Note that a mass of 1 kg weighs 9.8 N near the surface of Earth.)

Answer: 392 J

221. Find the work done when you push a box along the floor 2 m, when you apply a constant force of $F = 100$ N .

Answer: 200 J

222. Compute the work done for a force $F = 12/x^2$ N from $x = 1$ to $x = 2$ m.

Answer: 6 J

223. What is the work done moving a particle from $x = 0$ to $x = 1$ m if the force acting on it is $F = 3x^2$ N?

Answer: 1 J

For the following exercises, find the mass of the one-dimensional object.

224. A wire that is 2 ft long (starting at $x = 0$) and has a density function of $\rho(x) = x^2 + 2x$ lb/ft

Answer: $\frac{20}{3}$

225. A car antenna that is 3 ft long (starting at $x = 0$) and has a density function of $\rho(x) = 3x + 2$ lb/ft

Answer: $\frac{39}{2}$

226. A metal rod that is 8 in. long (starting at $x = 0$) and has a density function of $\rho(x) = e^{1/2x}$ lb/in.

Answer: $2(e^4 - 1)$)

227. A pencil that is 4 in. long (starting at $x = 2$) and has a density function of $\rho(x) = 5/x$ oz/in.

Answer: $\ln(243)$

228. A ruler that is 12 in. long (starting at $x = 5$) and has a density function of $\rho(x) = \ln(x) + (1/2)x^2$ oz/in.

Answer: $786 + 17\ln(17) - 5\ln(5)$

For the following exercises, find the mass of the two-dimensional object that is centered at the origin.

229. An oversized hockey puck of radius 2 in. with density function $\rho(x) = x^3 - 2x + 5$

Answer: $\dfrac{332\pi}{15}$

230. A frisbee of radius 6 in. with density function $\rho(x) = e^{-x}$

Answer: $\dfrac{2\pi}{e^6}\left(e^6 - 7\right)$

231. A plate of radius 10 in. with density function $\rho(x) = 1 + \cos(\pi x)$

Answer: 100π

232. A jar lid of radius 3 in. with density function $\rho(x) = \ln(x+1)$.

Answer: $\dfrac{\pi}{2}\left(4\ln(256) - 3\right)$

233. A disk of radius 5 cm with density function $\rho(x) = \sqrt{3x}$

Answer: $20\pi\sqrt{15}$

234. A 12-in. spring is stretched to 15 in. by a force of 75 lb. What is the spring constant?

Answer: $k = 300$ lb/ft

235. A spring has a natural length of 10 cm. It takes 2 J to stretch the spring to 15 cm. How much work would it take to stretch the spring from 15 cm to 20 cm?

Answer: 6 J

236. A 1-m spring requires 10 J to stretch the spring to 1.1 m. How much work would it take to stretch the spring from 1 m to 1.2 m?

Answer: 40 J

237. A spring requires 5J to stretch the spring from 8 cm to 12 cm, and an additional 4 J to stretch the spring from 12 cm to 14 cm. What is the natural length of the spring?

Answer: 5 cm

238. A shock absorber is compressed 1 in. by a weight of 1 t. What is the spring constant?

Answer: $k = 24,000$ lb/ft

239. A force of $F = 20x - x^3$ N stretches a nonlinear spring by x meters. What work is required to stretch the spring from $x = 0$ to $x = 2$ m?

Answer: 36 J

240. Find the work done by winding up a hanging cable of length 100 ft and weight-density 5 lb/ft.

Answer: $25,000$ ft-lb

241. For the cable in the preceding exercise, how much work is done to lift the cable 50 ft?

Answer: $18,750$ ft-lb

242. For the cable in the preceding exercise, how much additional work is done by hanging a 200 lb weight at the end of the cable?

Answer: $20,000$ ft-lb

243. **[T]** A pyramid of height 500 ft has a square base 800 ft by 800 ft. Find the area A at height h. If the rock used to build the pyramid weighs approximately $w = 100$ lb/ft^3, how much work did it take to lift all the rock?

Answer: $\frac{32}{3} \times 10^9$ ft-lb

244. **[T]** For the pyramid in the preceding exercise, assume there were 1000 workers each working 10 hours a day, 5 days a week, 50 weeks a year. If the workers, on average, lifted 10 100 lb rocks 2 ft/hr, how long did it take to build the pyramid?

Answer: 2 years 1 month 18 days 14 hours 24 minutes

245. **[T]** The force of gravity on a mass m is $F = -\left((GMm)/x^2\right)$ newtons. For a rocket of mass $m = 1000$ kg, compute the work to lift the rocket from $x = 6400$ to $x = 6500$ m. State your answer with three significant figures. (Note: $G = 6 \times 10^{-17}$ N m^2 / kg^2 and $M = 6 \times 10^{24}$ kg.)

Answer: 8.65×10^2 N m

246. **[T]** For the rocket in the preceding exercise, find the work to lift the rocket from $x = 6400$ to $x = \infty$.

Answer: 5.625×10^7 J

247. **[T]** A rectangular dam is 40 ft high and 60 ft wide. Compute the total force F on the dam when
   a. the surface of the water is at the top of the dam and
   b. the surface of the water is halfway down the dam.

Answer a. $3{,}000{,}000$ lb, b. $749{,}000$ lb

248. **[T]** Find the work required to pump all the water out of a cylinder that has a circular base of radius 5 ft and height 200 ft. Use the fact that the density of water is 62 lb/ft^3.

Answer: 31π million ft-lb

249. **[T]** Find the work required to pump all the water out of the cylinder in the preceding exercise if the cylinder is only half full.

Answer: 23.25π million ft-lb

250. **[T]** How much work is required to pump out a swimming pool if the area of the base is 800 ft^2, the water is 4 ft deep, and the top is 1 ft above the water level? Assume that the density of water is 62 lb/ft^3.

Answer: $595{,}200$ ft-lb

251. A cylinder of depth H and cross-sectional area A stands full of water at density ρ. Compute the work to pump all the water to the top.

Answer: $\dfrac{A\rho H^2}{2}$

252. For the cylinder in the preceding exercise, compute the work to pump all the water to the top if the cylinder is only half full.

Answer: $\dfrac{3A\rho H^2}{8}$

253. A cone-shaped tank has a cross-sectional area that increases with its depth: $A = \left(\pi r^2 h^2\right) / H^3$. Show that the work to empty it is half the work for a cylinder with the same height and base.

Answer: Answers may vary.