

Chapter 6
Applications of Integration
6.7 Integrals, Exponential Functions, and Logarithms

Section Exercises

For the following exercises, find the derivative $\frac{dy}{dx}$.

295. $y = \ln(2x)$

Answer: $\frac{1}{x}$

296. $y = \ln(2x + 1)$

Answer: $\frac{2}{2x + 1}$

297. $y = \frac{1}{\ln x}$

Answer: $-\frac{1}{x(\ln x)^2}$

For the following exercises, find the indefinite integral.

298. $\int \frac{dt}{3t}$

Answer: $\frac{\ln t}{3} + C$

299. $\int \frac{dx}{1+x}$

Answer: $\ln(x+1) + C$

For the following exercises, find the derivative dy/dx . (You can use a calculator to plot the function and the derivative to confirm that it is correct.)

300. [T] $y = \frac{\ln x}{x}$

Answer: $\frac{1 - \ln x}{x^2}$

301. [T] $y = x \ln x$

Answer: $\ln x + 1$

302. [T] $y = \log_{10} x$

Answer: $\frac{1}{x \ln(10)}$

303. [T] $y = \ln(\sin x)$

Answer: $\cot(x)$

304. [T] $y = \ln(\ln x)$

Answer: $\frac{1}{x \ln x}$

305. [T] $y = 7 \ln(4x)$

Answer: $\frac{7}{x}$

306. [T] $y = \ln((4x)^7)$

Answer: $\frac{7}{x}$

307. [T] $y = \ln(\tan x)$

Answer: $\csc(x) \sec x$

308. [T] $y = \ln(\tan(3x))$

Answer: $3 \csc(3x) \sec(3x)$

309. [T] $y = \ln(\cos^2 x)$

Answer: $-2 \tan x$

For the following exercises, find the definite or indefinite integral.

310. $\int_0^1 \frac{dx}{3+x}$

Answer: $\ln\left(\frac{4}{3}\right)$

311. $\int_0^1 \frac{dt}{3+2t}$

Answer: $\frac{1}{2} \ln\left(\frac{5}{3}\right)$

$$312. \quad \int_0^2 \frac{x \, dx}{x^2 + 1}$$

$$\text{Answer: } \frac{1}{2} \ln(5)$$

$$313. \quad \int_0^2 \frac{x^3 \, dx}{x^2 + 1}$$

$$\text{Answer: } 2 - \frac{1}{2} \ln(5)$$

$$314. \quad \int_2^e \frac{dx}{x \ln x}$$

$$\text{Answer: } -\ln(\ln 2)$$

$$315. \quad \int_2^e \frac{dx}{(x \ln(x))^2}$$

$$\text{Answer: } \frac{1}{\ln(2)} - 1$$

$$316. \quad \int \frac{\cos x \, dx}{\sin x}$$

$$\text{Answer: } \ln(\sin x) + C$$

$$317. \quad \int_0^{\pi/4} \tan x \, dx$$

$$\text{Answer: } \frac{1}{2} \ln(2)$$

$$318. \quad \int \cot(3x) \, dx$$

$$\text{Answer: } \frac{1}{3} \ln(\sin(3x)) + C$$

$$319. \quad \int \frac{(\ln x)^2 \, dx}{x}$$

$$\text{Answer: } \frac{1}{3} (\ln x)^3$$

For the following exercises, compute dy/dx by differentiating $\ln y$.

320. $y = \sqrt{x^2 + 1}$

Answer: $\frac{x}{\sqrt{x^2 + 1}}$

321. $y = \sqrt{x^2 + 1}\sqrt{x^2 - 1}$

Answer: $\frac{2x^3}{\sqrt{x^2 + 1}\sqrt{x^2 - 1}}$

322. $y = e^{\sin x}$

Answer: $e^{\sin x} \cos x$

323. $y = x^{-1/x}$

Answer: $x^{-2-(1/x)} (\ln x - 1)$

324. $y = e^{(ex)}$

Answer: e^{ex+1}

325. $y = x^e$

Answer: ex^{e-1}

326. $y = x^{(ex)}$

Answer: $ex^{ex} (\ln x + 1)$

327. $y = \sqrt{x}\sqrt[3]{x}\sqrt[6]{x}$

Answer: 1

328. $y = x^{-1/\ln x}$

Answer: 0

329. $y = e^{-\ln x}$

Answer: $-\frac{1}{x^2}$

For the following exercises, evaluate by any method.

330. $\int_5^{10} \frac{dt}{t} - \int_{5x}^{10x} \frac{dt}{t}$

Answer: 0

$$331. \quad \int_1^{e^{\pi}} \frac{dx}{x} + \int_{-2}^{-1} \frac{dx}{x}$$

Answer: $\pi - \ln(2)$

$$332. \quad \frac{d}{dx} \int_x^1 \frac{dt}{t}$$

Answer: $-\frac{1}{x}$

$$333. \quad \frac{d}{dx} \int_x^{x^2} \frac{dt}{t}$$

Answer: $\frac{1}{x}$

$$334. \quad \frac{d}{dx} \ln(\sec x + \tan x)$$

Answer: $\sec x$

For the following exercises, use the function $\ln x$. If you are unable to find intersection points analytically, use a calculator.

335. Find the area of the region enclosed by $x = 1$ and $y = 5$ above $y = \ln x$.

Answer: $e^5 - 6$ units²

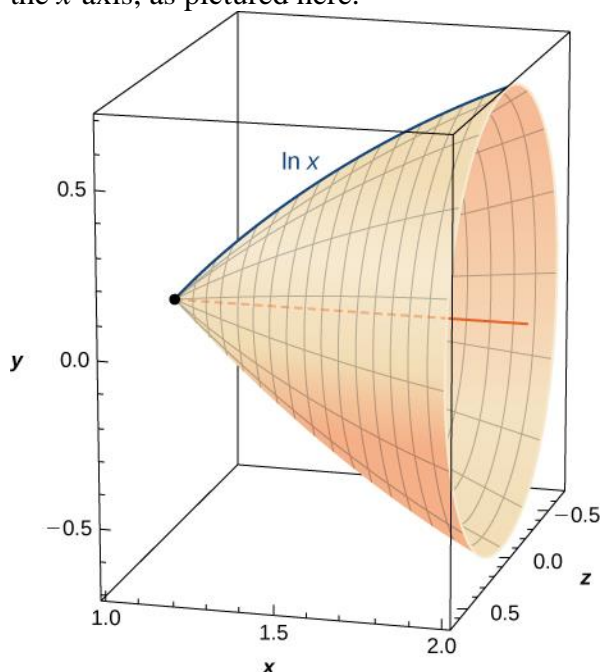
336. [T] Find the arc length of $\ln x$ from $x = 1$ to $x = 2$.

Answer: 1.22202

337. Find the area between $\ln x$ and the x -axis from $x = 1$ to $x = 2$.

Answer: $\ln(4) - 1$ units²

338. Find the volume of the shape created when rotating this curve from $x = 1$ to $x = 2$ around the x -axis, as pictured here.



Answer: $2\pi \left((\ln(2) - 1)^2 \right)$

339. [T] Find the surface area of the shape created when rotating the curve in the previous exercise from $x = 1$ to $x = 2$ around the x -axis.

Answer: 2.8656

If you are unable to find intersection points analytically in the following exercises, use a calculator.

340. Find the area of the hyperbolic quarter-circle enclosed by $x = 2$ and $y = 2$ above $y = 1/x$.

Answer: $3 - \ln(4)$

341. [T] Find the arc length of $y = 1/x$ from $x = 1$ to $x = 4$.

Answer: 3.1502

342. Find the area under $y = 1/x$ and above the x -axis from $x = 1$ to $x = 4$.

Answer: $\ln(4)$

For the following exercises, verify the derivatives and antiderivatives.

343.
$$\frac{d}{dx} \ln(x + \sqrt{x^2 + 1}) = \frac{1}{\sqrt{1 + x^2}}$$

Answer: This is a proof; therefore, no answer is provided.

$$344. \quad \frac{d}{dx} \ln \left(\frac{x-a}{x+a} \right) = \frac{2a}{(x^2 - a^2)}$$

Answer: This is a proof; therefore, no answer is provided.

$$345. \quad \frac{d}{dx} \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right) = -\frac{1}{x\sqrt{1-x^2}}$$

Answer: This is a proof; therefore, no answer is provided.

$$346. \quad \frac{d}{dx} \ln \left(x + \sqrt{x^2 - a^2} \right) = \frac{1}{\sqrt{x^2 - a^2}}$$

Answer: This is a proof; therefore, no answer is provided.

$$347. \quad \int \frac{dx}{x \ln(x) \ln(\ln x)} = \ln(\ln(\ln x)) + C$$

Answer: This is a proof; therefore, no answer is provided.

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