Chapter 6
Applications of Integration
6.8 Exponential Growth and Decay

Section Exercises
***True or False*? If true, prove it. If false, find the true answer.**

348. The doubling time for $y = e^{ct}$ is $\left(\ln(2)\right)/\left(\ln(c)\right)$.

Answer: False; $t = \frac{\ln(2)}{c}$

349. If you invest $\$500$, an annual rate of interest of 3% yields more money in the first year than a 2.5% continuous rate of interest.

Answer: True

350. If you leave a 100 °C pot of tea at room temperature (25 °C) and an identical pot in the refrigerator (5 °C), with $k = 0.02,$ the tea in the refrigerator reaches a drinkable temperature (70 °C) more than 5 minutes before the tea at room temperature.

Answer: True

351. If given a half-life of t years, the constant k for $y = e^{kt}$ is calculated by $k = \ln\left(1/2\right)/t$.

Answer: False; $k = \frac{\ln(2)}{t}$

For the following exercises, use $y = y_0 e^{kt}$.

352. If a culture of bacteria doubles in 3 hours, how many hours does it take to multiply by 10?

Answer: 9 hours 58 minutes

353. If bacteria increase by a factor of 10 in 10 hours, how many hours does it take to increase by 100?

Answer: 20 hours

354. How old is a skull that contains one-fifth as much radiocarbon as a modern skull? Note that the half-life of radiocarbon is 5730 years.

Answer: $\frac{5730\ln(5)}{\ln(2)}$ or about $13,300$ years

355. If a relic contains 90% as much radiocarbon as new material, can it have come from the time of Christ (approximately 2000 years ago)? Note that the half-life of radiocarbon is 5730 years.

Answer: No. The relic is approximately 871 years old.

356. The population of Cairo grew from 5 million to 10 million in 20 years. Use an exponential model to find when the population was 8 million.

Answer: $\dfrac{20\left(3\ln(2)-\ln(5)\right)}{\ln(2)}$, about 13.56 years in

357. The populations of New York and Los Angeles are growing at 1% and 1.4% a year, respectively. Starting from 8 million (New York) and 6 million (Los Angeles), when are the populations equal?

Answer: 71.92 years

358. Suppose the value of $\$1$ in Japanese yen decreases at 2% per year. Starting from $\$1$ = ¥250, when will $\$1 =$ ¥1?

Answer: 273 years from now

359. The effect of advertising decays exponentially. If 40% of the population remembers a new product after 3 days, how long will 20% remember it?

Answer: 5 days 6 hours 27 minutes

360. If $y = 1000$ at $t = 3$ and $y = 3000$ at $t = 4$, what was y_0 at $t = 0$?

Answer: $\dfrac{1000}{27}$

361. If $y = 100$ at $t = 4$ and $y = 10$ at $t = 8$, when does $y = 1$?

Answer: 12

362. If a bank offers annual interest of 7.5% or continuous interest of 7.25%, which has a better annual yield?

Answer: Continuous interest of 7.5%

363. What continuous interest rate has the same yield as an annual rate of 9%?

Answer: 8.618%

364. If you deposit $\$5000$ at 8% annual interest, how many years can you withdraw $\$500$ (starting after the first year) without running out of money?

Answer: 20 years. You will be unable to withdraw a full $\$500$ the 21st year.

365. You are trying to save $\$50,000$ in 20 years for college tuition for your child. If interest is a continuous 10%, how much do you need to invest initially?

Answer: $\$6766.76$

366. You are cooling a turkey that was taken out of the oven with an internal temperature of $165°\text{F}.$ After 10 minutes of resting the turkey in a $70°\text{F}$ apartment, the temperature has reached $155°\text{F}.$ What is the temperature of the turkey 20 minutes after taking it out of the oven?

Answer: $146.05°\text{F}$

367. You are trying to thaw some vegetables that are at a temperature of $1°\text{F}.$ To thaw vegetables safely, you must put them in the refrigerator, which has an ambient temperature of $44°\text{F}.$ You check on your vegetables 2 hours after putting them in the refrigerator to find that they are now $12°\text{F}.$ Plot the resulting temperature curve and use it to determine when the vegetables reach $33°\text{F}.$

Answer: 9 hours 13 minutes

368. You are an archaeologist and are given a bone that is claimed to be from a Tyrannosaurus Rex. You know these dinosaurs lived during the Cretaceous Era (146 million years to 65 million years ago), and you find by radiocarbon dating that there is 0.000001% the amount of radiocarbon. Is this bone from the Cretaceous?

Answer: No. The bone is only $114,208$ years old.

369. The spent fuel of a nuclear reactor contains plutonium-239, which has a half-life of $24,000$ years. If 1 barrel containing 10 kg of plutonium-239 is sealed, how many years must pass until only $10\ g$ of plutonium-239 is left?

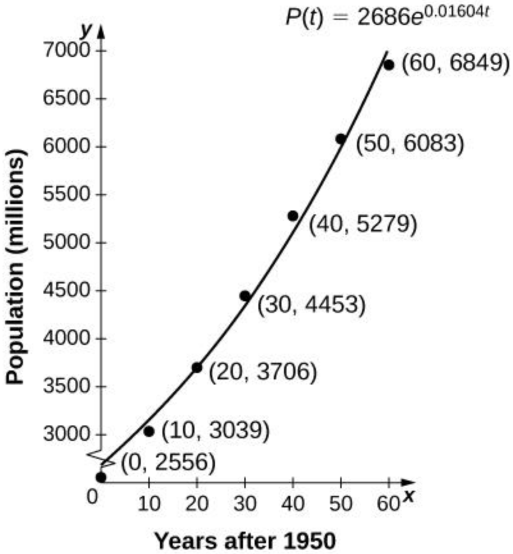
Answer: $239,179$ years

For the next set of exercises, use the following table, which features the world population by decade.

Years since 1950	**Population (millions)**
0	2,556
10	3,039
20	3,706
30	4,453
40	5,279
50	6,083
60	6,849

370. **[T]** The best-fit exponential curve to the data of the form $P(t) = ae^{bt}$ is given by $P(t) = 2686e^{0.01604t}$. Use a graphing calculator to graph the data and the exponential curve together.

Answer:



371. **[T]** Find and graph the derivative y' of your equation. Where is it increasing and what is the meaning of this increase?

Answer: $P'(t) = 43e^{0.01604t}$. The population is always increasing.

372. **[T]** Find and graph the second derivative of your equation. Where is it increasing and what is the meaning of this increase?

Answer: $P''(t) = 0.6911e^{0.01604t}$. The rate at which the population increases is growing; population growth is always accelerating.

373. **[T]** Find the predicted date when the population reaches 10 billion. Using your previous answers about the first and second derivatives, explain why exponential growth is unsuccessful in predicting the future.

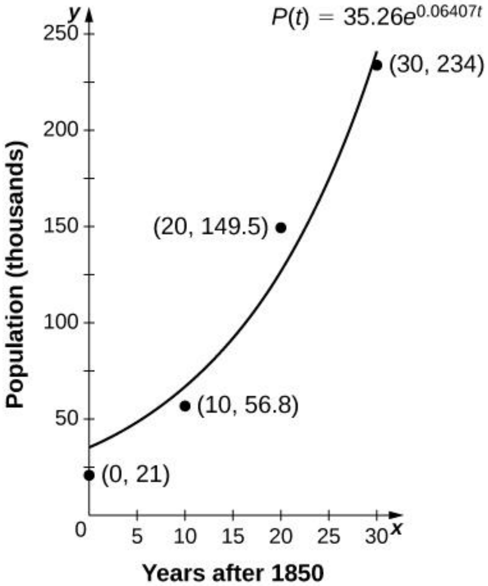
Answer: The population reaches 10 billion people in 2027.

For the next set of exercises, use the following table, which shows the population of San Francisco during the 19th century.

Years since 1850	**Population (thousands)**
0	21.00
10	56.80
20	149.5
30	234.0

374. [**T**] The best-fit exponential curve to the data of the form $P(t) = ae^{bt}$ is given by $P(t) = 35.26e^{0.06407t}$. Use a graphing calculator to graph the data and the exponential curve together.

Answer:



375. [**T**] Find and graph the derivative y' of your equation. Where is it increasing? What is the meaning of this increase? Is there a value where the increase is maximal?

Answer: $P'(t) = 2.259e^{0.06407t}$. The population is always increasing.

376. [**T**] Find and graph the second derivative of your equation. Where is it increasing? What is the meaning of this increase?

Answer: $P''(t) = 0.1447e^{0.06407t}$. The rate at which the population increases is growing, so population growth is accelerating.