

**Chapter 5****Integration****5.4. Integration Formulas and the Net Change Theorem****Section Exercises**

Use basic integration formulas to compute the following antiderivatives or definite intervals.

$$207. \quad \int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$$

$$\text{Answer: } \int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) dx = \int x^{1/2} dx - \int x^{-1/2} dx = \frac{2}{3} x^{3/2} + C_1 - 2x^{1/2} + C_2 = \frac{2}{3} x^{3/2} - 2x^{1/2} + C$$

$$208. \quad \int \left( e^{2x} - \frac{1}{2} e^{x/2} \right) dx$$

$$\text{Answer: } \int \left( e^{2x} - \frac{1}{2} e^{x/2} \right) dx = \int e^{2x} dx - \int \frac{1}{2} e^{x/2} dx = \frac{e^{2x}}{2} + C_1 - e^{x/2} + C_2 = \frac{e^{2x}}{2} - e^{x/2} + C$$

$$209. \quad \int \frac{dx}{2x}$$

$$\text{Answer: } \int \frac{dx}{2x} = \frac{1}{2} \ln|x| + C$$

$$210. \quad \int \frac{x-1}{x^2} dx$$

$$\text{Answer: } \int \frac{x-1}{x^2} dx = \int \frac{dx}{x} - \int \frac{dx}{x^2} = \ln|x| + C_1 - \left( -\frac{1}{x} + C_2 \right) = \ln|x| + \frac{1}{x} + C$$

$$211. \quad \int_0^{\pi} (\sin x - \cos x) dx$$

$$\text{Answer: } \int_0^{\pi} \sin x dx - \int_0^{\pi} \cos x dx = -\cos x \Big|_0^{\pi} - (\sin x) \Big|_0^{\pi} = -(-1) + 1 - (0 - 0) = 2$$

$$212. \quad \int_0^{\pi/2} (x - \sin x) dx$$

$$\text{Answer: } \int_0^{\pi/2} x dx - \int_0^{\pi/2} \sin x dx = \left( \frac{x^2}{2} \right) \Big|_0^{\pi/2} + \cos x \Big|_0^{\pi/2} = \frac{\pi^2}{8} - 1$$

213. Write an integral that expresses the increase in the perimeter  $P(s)$  of a square when its side length  $s$  increases from 2 units to 4 units and evaluate the integral.

Answer:  $P(s) = 4s$ , so  $\frac{dP}{ds} = 4$  and  $\int_2^4 4ds = 8$ .

214. Write an integral that quantifies the change in the area  $A(s) = s^2$  of a square when the side length doubles from  $S$  units to  $2S$  units and evaluate the integral.

Answer:  $A(s) = s^2$ , so  $\frac{dA}{ds} = 2s$  and  $\int_S^{2S} 2sds = 4S^2 - S^2 = 3S^2$ .

215. A regular  $N$ -gon (an  $N$ -sided polygon with sides that have equal length  $s$ , such as a pentagon or hexagon) has perimeter  $Ns$ . Write an integral that expresses the increase in perimeter of a regular  $N$ -gon when the length of each side increases from 1 unit to 2 units and evaluate the integral.

Answer:  $\int_1^2 Nds = N$

216. The area of a regular pentagon with side length  $a > 0$  is  $pa^2$  with  $p = \frac{1}{4}\sqrt{5 + \sqrt{5 + 2\sqrt{5}}}$ .

The Pentagon in Washington, DC, has inner sides of length 360 ft and outer sides of length 920 ft. Write an integral to express the area of the roof of the Pentagon according to these dimensions and evaluate this area.

Answer:  $A = \int_{360}^{920} 2pa da = pa^2 \Big|_{360}^{920} = p(560)(1280) = 179,200\sqrt{5 + \sqrt{5 + 2\sqrt{5}}} = 509,309 \text{ ft}^2$

217. A dodecahedron is a Platonic solid with a surface that consists of 12 pentagons, each of equal area. By how much does the surface area of a dodecahedron increase as the side length of each pentagon doubles from 1 unit to 2 units?

Answer: With  $p$  as in the previous exercise, each of the 12 pentagons increases in area from  $2p$  to  $4p$  units so the net increase in the area of the dodecahedron is  $36p$  units.

218. An icosahedron is a Platonic solid with a surface that consists of 20 equilateral triangles. By how much does the surface area of an icosahedron increase as the side length of each triangle doubles from  $a$  unit to  $2a$  units?

Answer:  $15\sqrt{3}a^2$

219. Write an integral that quantifies the change in the area of the surface of a cube when its side length doubles from  $s$  unit to  $2s$  units and evaluate the integral.

Answer:  $18s^2 = 6\int_s^{2s} 2xdx$

220. Write an integral that quantifies the increase in the volume of a cube when the side length doubles from  $s$  unit to  $2s$  units and evaluate the integral.

Answer:  $7s^3 = \int_s^{2s} 3x^2 dx$

221. Write an integral that quantifies the increase in the surface area of a sphere as its radius doubles from  $R$  unit to  $2R$  units and evaluate the integral.

Answer:  $12\pi R^2 = 8\pi \int_R^{2R} r dr$

222. Write an integral that quantifies the increase in the volume of a sphere as its radius doubles from  $R$  unit to  $2R$  units and evaluate the integral.

Answer:  $\frac{28\pi R^3}{3} = 4\pi \int_R^{2R} r^2 dr$

223. Suppose that a particle moves along a straight line with velocity  $v(t) = 4 - 2t$ , where  $0 \leq t \leq 2$  (in meters per second). Find the displacement at time  $t$  and the total distance traveled up to  $t = 2$ .

Answer:  $d(t) = \int_0^t v(s) ds = 4t - t^2$ . The total distance is  $d(2) = 4$  m.

224. Suppose that a particle moves along a straight line with velocity defined by  $v(t) = t^2 - 3t - 18$ , where  $0 \leq t \leq 6$  (in meters per second). Find the displacement at time  $t$  and the total distance traveled up to  $t = 6$ .

Answer:  $d(t) = \int_0^t v(s) ds = \frac{t^3}{3} - \frac{3t^2}{2} - 18t$ . The total distance traveled is 90 m.

225. Suppose that a particle moves along a straight line with velocity defined by  $v(t) = |2t - 6|$ , where  $0 \leq t \leq 6$  (in meters per second). Find the displacement at time  $t$  and the total distance traveled up to  $t = 6$ .

Answer:  $d(t) = \int_0^t v(s) ds$ . For  $t < 3$ ,  $d(t) = \int_0^t (6 - 2t) dt = 6t - t^2$ . For  $t > 3$ ,

$$d(t) = d(3) + \int_3^t (2t - 6) dt$$

$$\text{So, } d(6) = d(3) + \int_3^6 (2t - 6) dt = 9 + \left( t^2 - 6t \right) \Big|_3^6$$

$$= 9 + (36 - 36) - (9 - 18) = 9 + 9 = 18$$

The total distance is  $d(6) = 18$  m.

226. Suppose that a particle moves along a straight line with acceleration defined by  $a(t) = t - 3$ , where  $0 \leq t \leq 6$  (in meters per second). Find the velocity and displacement at time  $t$  and the total distance traveled up to  $t = 6$  if  $v(0) = 3$  and  $d(0) = 0$ .

Answer:  $v(t) = v(0) + \int_0^t a(t) dt = 3 + \frac{t^2}{2} - 3t$  and  $d(t) = \int_0^t v(t) dt = 3t + \frac{t^3}{6} - \frac{3t^2}{2}$ . When  $t = 6$ ,  $d(6) = 0$ .

$$v(t) > 0 \text{ on } [0, 3 - \sqrt{3}] \text{ and } [3 + \sqrt{3}, 6]; v(t) < 0 \text{ on } [3 - \sqrt{3}, 3 + \sqrt{3}].$$

$$\begin{aligned} &= \int_0^{3-\sqrt{3}} \left(3 + \frac{t^2}{2} - 3t\right) dt - \int_{3-\sqrt{3}}^{3+\sqrt{3}} \left(3 + \frac{t^2}{2} - 3t\right) dt + \int_{3+\sqrt{3}}^6 \left(3 + \frac{t^2}{2} - 3t\right) dt \\ \text{Total displacement} &= \left[3t + \frac{t^3}{6} - \frac{3t^2}{2}\right]_0^{3-\sqrt{3}} - \left[3t + \frac{t^3}{6} - \frac{3t^2}{2}\right]_{3-\sqrt{3}}^{3+\sqrt{3}} + \left[3t + \frac{t^3}{6} - \frac{3t^2}{2}\right]_{3+\sqrt{3}}^6 \\ &= (\sqrt{3} - 0) - (-\sqrt{3} - \sqrt{3}) + (0 + \sqrt{3}) \\ &= 4\sqrt{3} \end{aligned}$$

227. A ball is thrown upward from a height of 1.5 m at an initial speed of 40 m/sec. Acceleration resulting from gravity is  $-9.8 \text{ m/sec}^2$ . Neglecting air resistance, solve for the velocity  $v(t)$  and the height  $h(t)$  of the ball  $t$  seconds after it is thrown and before it returns to the ground.

Answer:  $v(t) = 40 - 9.8t \text{ m/sec}$ ;  $h(t) = 1.5 + 40t - 4.9t^2 \text{ m}$

228. A ball is thrown upward from a height of 3 m at an initial speed of 60 m/sec. Acceleration resulting from gravity is  $-9.8 \text{ m/sec}^2$ . Neglecting air resistance, solve for the velocity  $v(t)$  and the height  $h(t)$  of the ball  $t$  seconds after it is thrown and before it returns to the ground.

Answer:  $v(t) = 60 - 9.8t$ ;  $h(t) = 3 + 60t - 4.9t^2 \text{ m/s}$

229. The area  $A(t)$  of a circular shape is growing at a constant rate. If the area increases from  $4\pi$  units to  $9\pi$  units between times  $t = 2$  and  $t = 3$ , find the net change in the radius during that time.

Answer: The net increase is 1 unit.

230. A spherical balloon is being inflated at a constant rate. If the volume of the balloon changes from  $36\pi \text{ in.}^3$  to  $288\pi \text{ in.}^3$  between time  $t = 30$  and  $t = 60$  seconds, find the net change in the radius of the balloon during that time.

Answer:  $r(30) = \left(\frac{3V(30)}{4\pi}\right)^{1/3} = 3$  and  $r(60) = \left(\frac{3V(60)}{4\pi}\right)^{1/3} = 6$  so the net increase in radius is 3 units.

231. Water flows into a conical tank with cross-sectional area  $\pi x^2$  at height  $x$  and volume  $\frac{\pi x^3}{3}$  up to height  $x$ . If water flows into the tank at a rate of  $1 \text{ m}^3/\text{min}$ , find the height of water in the tank after 5 min. Find the change in height between 5 min and 10 min.

Answer: At  $t = 5$ , the height of water is  $x = \left(\frac{15}{\pi}\right)^{1/3} \text{ m}$ . The net change in height from  $t = 5$  to  $t = 10$  is  $\left(\frac{30}{\pi}\right)^{1/3} - \left(\frac{15}{\pi}\right)^{1/3} \text{ m}$ .

232. A horizontal cylindrical tank has cross-sectional area  $A(x) = 4(6x - x^2) \text{ m}^2$  at height  $x$  meters above the bottom when  $x \leq 3$ .
- The volume  $V$  between heights  $a$  and  $b$  is  $\int_a^b A(x) dx$ . Find the volume at heights between 2 m and 3 m.
  - Suppose that oil is being pumped into the tank at a rate of  $50 \text{ L/min}$ . Using the chain rule,  $\frac{dx}{dt} = \frac{dx}{dV} \frac{dV}{dt}$ , at how many meters per minute is the height of oil in the tank changing, expressed in terms of  $x$ , when the height is at  $x$  meters?
  - How long does it take to fill the tank to 3 m starting from a fill level of 2 m?

Answer: a.  $V = 4 \int_2^3 (6s - s^2) ds = 4 \left( 3s^2 - \frac{s^3}{3} \right) \Big|_2^3 = \frac{104}{3} \text{ m}^3$ ; b.  $\frac{dx}{dt} = \frac{0.05}{4(6x - x^2)} \text{ m/min}$ ; c.

$$\frac{104}{3 \cdot 0.05} = 693.33 \text{ min}$$

233. The following table lists the electrical power in gigawatts—the rate at which energy is consumed—used in a certain city for different hours of the day, in a typical 24-hour period, with hour 1 corresponding to midnight to 1 a.m.

Hour	Power	Hour	Power
1	28	13	48
2	25	14	49
3	24	15	49
4	23	16	50
5	24	17	50
6	27	18	50
7	29	19	46
8	32	20	43
9	34	21	42
10	39	22	40
11	42	23	37
12	46	24	34

Find the total amount of power in gigawatt-hours (gW-h) consumed by the city in a typical 24-hour period.

Answer: The total daily power consumption is estimated as the sum of the hourly power rates, or 911 gW-h.

234. The average residential electrical power use (in hundreds of watts) per hour is given in the following table.

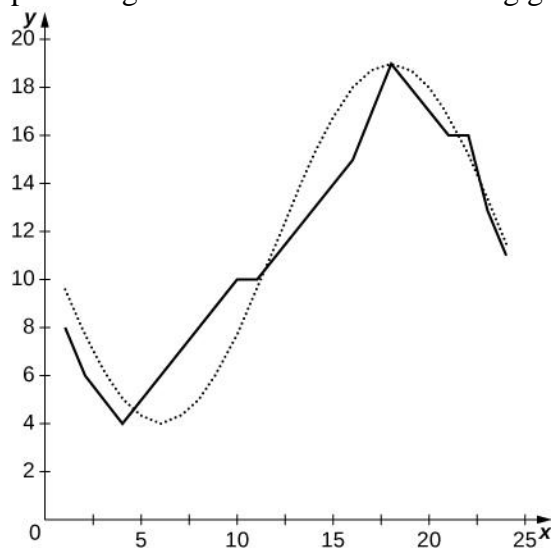
Hour	Power	Hour	Power
1	8	13	12
2	6	14	13
3	5	15	14
4	4	16	15
5	5	17	17
6	6	18	19
7	7	19	18
8	8	20	17
9	9	21	16
10	10	22	16
11	10	23	13
12	11	24	11

- Compute the average total energy used in a day in kilowatt-hours (kWh).
- If a ton of coal generates 1842 kWh, how long does it take for an average residence to burn a ton of coal?
- Explain why the data might fit a plot of the form  $p(t) = 11.5 - 7.5 \sin\left(\frac{\pi t}{12}\right)$ .

Answer: a. The energy usage in hour  $n$  is estimated as the reported energy rate for hour  $n$  times the duration, 1 hour. The average total energy is then the sum of the numbers in the “power”

rows divided by 10 for the units conversion, or 27 kWh. b.  $\frac{1842}{27} = 68$  days. c. The daily power

use might be sinusoidal because of different levels of activity at different times of day.  $p(t)$  is plotted against the data in the following graph.



235. The data in the following table are used to estimate the average power output produced by Peter Sagan for each of the last 18 sec of Stage 1 of the 2012 Tour de France.

Average Power Output

Second	Watts	Second	Watts
1	600	10	1200
2	500	11	1170
3	575	12	1125
4	1050	13	1100
5	925	14	1075
6	950	15	1000
7	1050	16	950
8	950	17	900
9	1100	18	780

Estimate the net energy used in kilojoules (kJ), noting that  $1\text{W} = 1\text{ J/s}$ , and the average power output by Sagan during this time interval.

Answer: 17 kJ

236. The data in the following table are used to estimate the average power output produced by Peter Sagan for each 15-min interval of Stage 1 of the 2012 Tour de France.

Average Power Output

Minutes	Watts	Minutes	Watts
15	200	165	170
30	180	180	220
45	190	195	140
60	230	210	225
75	240	225	170
90	210	240	210
105	210	255	200
120	220	270	220
135	210	285	250
150	150	300	400

Estimate the net energy used in kilojoules, noting that  $1\text{W} = 1\text{ J/s}$ .

Answer: 3820 kJ

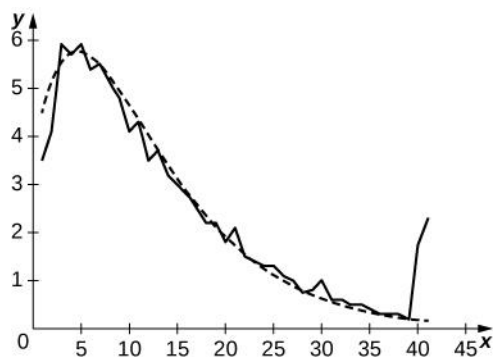
237. The distribution of incomes as of 2012 in the United States in \$5000 increments is given in the following table. The  $k$ th row denotes the percentage of households with incomes between  $\$5000xk$  and  $5000xk + 4999$ . The row  $k = 40$  contains all households with income between \$200,000 and \$250,000 and  $k = 41$  accounts for all households with income exceeding \$250,000.

Income Distributions

0	3.5	21	1.5
1	4.1	22	1.4
2	5.9	23	1.3
3	5.7	24	1.3
4	5.9	25	1.1
5	5.4	26	1.0
6	5.5	27	0.75
7	5.1	28	0.8
8	4.8	29	1.0
9	4.1	30	0.6
10	4.3	31	0.6
11	3.5	32	0.5
12	3.7	33	0.5
13	3.2	34	0.4
14	3.0	35	0.3
15	2.8	36	0.3
16	2.5	37	0.3
17	2.2	38	0.2
18	2.2	39	1.8
19	1.8	40	2.3
20	2.1	41	

- Estimate the percentage of U.S. households in 2012 with incomes less than \$55,000.
- What percentage of households had incomes exceeding \$85,000?
- Plot the data and try to fit its shape to that of a graph of the form  $a(x+c)e^{-b(x+c)}$  for suitable  $a, b, c$ .

Answer: a. 54.3%; b. 27.00%; c. The curve in the following plot is  $2.35(t+3)e^{-0.15(t+3)}$ .





238. Newton's law of gravity states that the gravitational force exerted by an object of mass  $M$  and one of mass  $m$  with centers that are separated by a distance  $r$  is  $F = G \frac{mM}{r^2}$ , with  $G$  an empirical constant  $G = 6.67 \times 10^{-11} \text{ m}^3 / (\text{kg} \cdot \text{s}^2)$ . The work done by a variable force over an interval  $[a, b]$  is defined as  $W = \int_a^b F(x) dx$ . If Earth has mass  $5.97219 \times 10^{24}$  and radius 6371 km, compute the amount of work to elevate a polar weather satellite of mass 1400 kg to its orbiting altitude of 850 km above Earth.

Answer:  $W = GmM \int_{6371}^{7221} \frac{1}{r^2} dr = GmM \left[ -\frac{1}{r} \right]_{6371}^{7221} = 6.67 \times 10^{-11} \cdot 5.972 \times 10^{24} \cdot 1.4 \times 10^3 \cdot 10^{-3} \left( \frac{1}{6371} - \frac{1}{7221} \right) = 1.03 \times 10^{10} \text{ N}\cdot\text{m}$

239. For a given motor vehicle, the maximum achievable deceleration from braking is approximately  $7 \text{ m/sec}^2$  on dry concrete. On wet asphalt, it is approximately  $2.5 \text{ m/sec}^2$ . Given that 1 mph corresponds to 0.447 m/sec, find the total distance that a car travels in meters on dry concrete after the brakes are applied until it comes to a complete stop if the initial velocity is 67 mph (30 m/sec) or if the initial braking velocity is 56 mph (25 m/sec). Find the corresponding distances if the surface is slippery wet asphalt.

Answer: In dry conditions, with initial velocity  $v_0 = 30 \text{ m/s}$ ,  $D = 64.3$  and, if  $v_0 = 25$ ,  $D = 44.64$ . In wet conditions, if  $v_0 = 30$ , and  $D = 180$  and if  $v_0 = 25$ ,  $D = 125$ .

240. John is a 25-year old man who weighs 160 lb. He burns  $500 - 50t$  calories/hr while riding his bike for  $t$  hours. If an oatmeal cookie has 55 cal and John eats  $4t$  cookies during the  $t$ th hour, how many net calories has he lost after 3 hours riding his bike?

Answer: 285 cal

241. Sandra is a 25-year old woman who weighs 120 lb. She burns  $300 - 50t$  cal/hr while walking on her treadmill. Her caloric intake from drinking Gatorade is  $100t$  calories during the  $t$ th hour. What is her net decrease in calories after walking for 3 hours?

Answer: 225 cal

242. A motor vehicle has a maximum efficiency of 33 mpg at a cruising speed of 40 mph. The efficiency drops at a rate of 0.1 mpg/mph between 40 mph and 50 mph, and at a rate of 0.4 mpg/mph between 50 mph and 80 mph. What is the efficiency in miles per gallon if the car is cruising at 50 mph? What is the efficiency in miles per gallon if the car is cruising at 80 mph? If gasoline costs \$3.50/gal, what is the cost of fuel to drive 50 mi at 40 mph, at 50 mph, and at 80 mph?

Answer: Efficiency:  $E(40) = 33$ ,  $E(50) = 32$ ,  $E(80) = 20$ ; Cost:  $C(40) = 5.30$ ,  $C(50) = 5.47$  and  $C(80) = 8.75$

243. Although some engines are more efficient at given a horsepower than others, on average, fuel efficiency decreases with horsepower at a rate of  $1/25$  mpg/horsepower. If a typical 50-horsepower engine has an average fuel efficiency of 32 mpg, what is the average fuel efficiency of an engine with the following horsepower: 150, 300, 450?

Answer:  $E(150) = 28$ ,  $E(300) = 22$ ,  $E(450) = 16$

244. [T] The following table lists the 2013 schedule of federal income tax versus taxable income.

Federal Income Tax Versus Taxable Income

<b>Taxable Income Range</b>	<b>The Tax Is ...</b>	<b>... Of the Amount Over</b>
\$0–\$8925	10%	\$0
\$8925– \$36,250	\$892.50 + 15%	\$8925
\$36,250– \$87,850	\$4,991.25 + 25%	\$36,250
\$87,850– \$183,250	\$17,891.25 + 28%	\$87,850
\$183,250– \$398,350	\$44,603.25 + 33%	\$183,250
\$398,350– \$400,000	\$115,586.25 + 35%	\$398,350
> \$400,000	\$116,163.75 + 39.6%	\$400,000

Suppose that Steve just received a \$10,000 raise. How much of this raise is left after federal taxes if Steve's salary before receiving the raise was \$40,000? If it was \$90,000? If it was \$385,000?

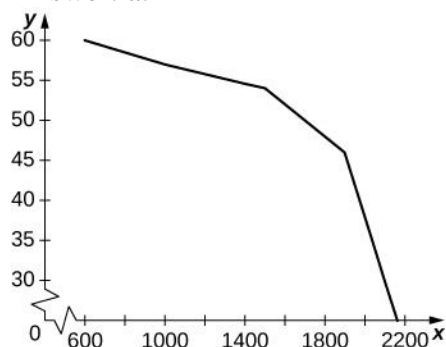
Answer: At \$40,000 the tax rate is 25%, so \$7,500 is left after taxes. At \$90,000 the tax rate is 28% and \$7,200 is left. At \$385,000 the tax rate is 35% and \$6,500 is left.

245. [T] The following table provides hypothetical data regarding the level of service for a certain highway.

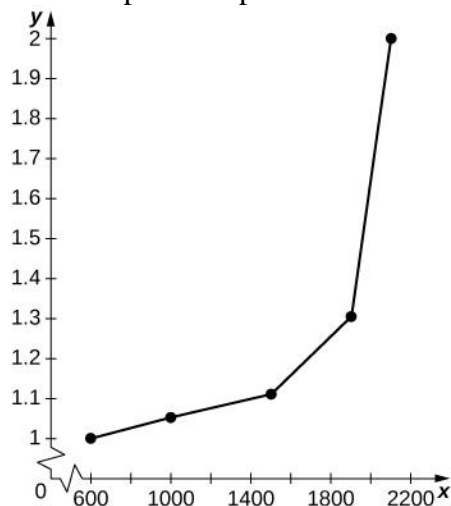
Highway Speed Range (mph)	Vehicles per Hour per Lane	Density Range (vehicles/mi)
> 60	< 600	< 10
60–57	600–1000	10–20
57–54	1000–1500	20–30
54–46	1500–1900	30–45
46–30	1900–2100	45–70
<30	Unstable	70–200

- Plot vehicles per hour per lane on the  $x$ -axis and highway speed on the  $y$ -axis.
- Compute the average decrease in speed (in miles per hour) per unit increase in congestion (vehicles per hour per lane) as the latter increases from 600 to 1000, from 1000 to 1500, and from 1500 to 2100. Does the decrease in miles per hour depend linearly on the increase in vehicles per hour per lane?
- Plot minutes per mile (60 times the reciprocal of miles per hour) as a function of vehicles per hour per lane. Is this function linear?

Answer: a.



b. Between 600 and 1000 the average decrease in vehicles per hour per lane is  $-0.0075$ . Between 1000 and 1500 it is  $-0.006$  per vehicles per hour per lane, and between 1500 and 2100 it is  $-0.04$  vehicles per hour per lane. c.

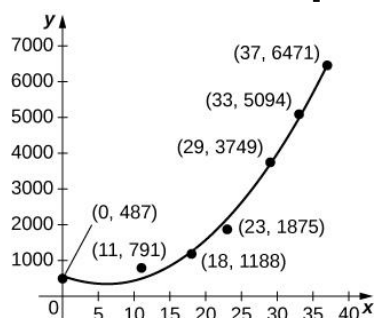


The graph is nonlinear, with minutes per mile increasing dramatically as vehicles per hour per lane reach 2000.

**For the next two exercises use the data in the following table, which displays bald eagle populations from 1963 to 2000 in the continental United States.**

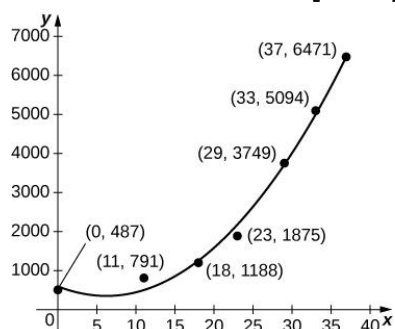
Year	Population of Breeding Pairs of Bald Eagles
1963	487
1974	791
1981	1188
1986	1875
1992	3749
1996	5094
2000	6471

246. [T] The graph below plots the quadratic  $p(t) = 6.48t^2 - 80.31t + 585.69$  against the data in preceding table, normalized so that  $t = 0$  corresponds to 1963. Estimate the average number of bald eagles per year present for the 37 years by computing the average value of  $p$  over  $[0, 37]$ .



$$\text{Answer: } \frac{1}{37} \int_0^{37} p(t) dt = \frac{6.48(37)^2}{3} - \frac{80.31(37)}{2} + 585.69 = 2057$$

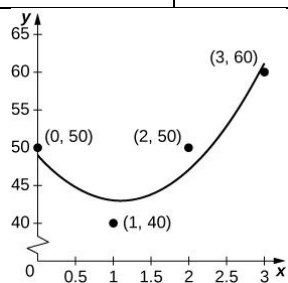
247. [T] The graph below plots the cubic  $p(t) = 0.07t^3 + 2.42t^2 - 25.63t + 521.23$  against the data in the preceding table, normalized so that  $t = 0$  corresponds to 1963. Estimate the average number of bald eagles per year present for the 37 years by computing the average value of  $p$  over  $[0, 37]$ .



Answer:  $\frac{1}{37} \int_0^{37} p(t) dt = \frac{0.07(37)^3}{4} + \frac{2.42(37)^2}{3} - \frac{25.63(37)}{2} + 521.23 \approx 2037$

248. [T] Suppose you go on a road trip and record your speed at every half hour, as compiled in the following table. The best quadratic fit to the data is  $q(t) = 5t^2 - 11t + 49$ , shown in the accompanying graph. Integrate  $q$  to estimate the total distance driven over the 3 hours.

Time (hr)	Speed (mph)
0 (start)	50
1	40
2	50
3	60

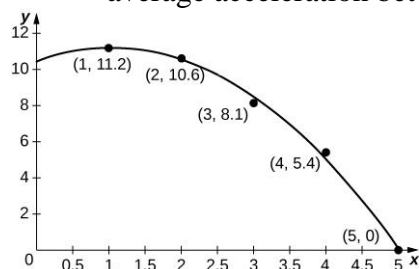


Answer:  $\int_0^3 q(t) dt = \frac{5(3)^3}{3} - \frac{11(3)^3}{2} + 49(3) = 142.5$  miles

As a car accelerates, it does not accelerate at a constant rate; rather, the acceleration is variable. For the following exercises, use the following table, which contains the acceleration measured at every second as a driver merges onto a freeway.

Time (sec)	Acceleration (mph/sec)
1	11.2
2	10.6
3	8.1
4	5.4
5	0

249. [T] The accompanying graph plots the best quadratic fit,  $a(t) = -0.70t^2 + 1.44t + 10.44$ , to the data from the preceding table. Compute the average value of  $a(t)$  to estimate the average acceleration between  $t = 0$  and  $t = 5$ .



Answer: Average acceleration is  $A = \frac{1}{5} \int_0^5 a(t) dt = -\frac{0.7(5^2)}{3} + \frac{1.44(5)}{2} + 10.44 \approx 8.2$  mph/s

250. [T] Using your acceleration equation from the previous exercise, find the corresponding velocity equation. Assuming the final velocity is 0 mph, find the velocity at time  $t = 0$ .

Answer:  $v(t) = -\frac{7}{30}t^3 + 0.72t^2 + 10.44t + C$ . Setting  $v(5) = 0$  gives  $C = -41.033$ . The initial speed is 41.03 mph. The sign is negative because the signs for the acceleration values are positive.

251. [T] Using your velocity equation from the previous exercise, find the corresponding distance equation, assuming your initial distance is 0 mi. How far did you travel while you accelerated your car? (Hint: You will need to convert time units.)

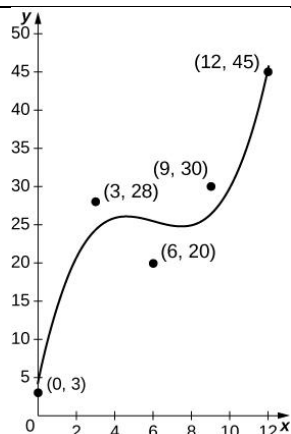
Answer:

$$d(t) = \int_0^t |v(t)| dt = \int_0^t \left( \frac{7}{30}t^3 - 0.72t^2 - 10.44t + 41.033 \right) dt = \frac{7}{120}t^4 - 0.24t^3 - 5.22t^2 + 41.033t.$$

Then,  $d(5) \approx 81.12 \text{ mph} \times \text{sec} \approx 119$  feet.

252. [T] The number of hamburgers sold at a restaurant throughout the day is given in the following table, with the accompanying graph plotting the best cubic fit to the data,  $b(t) = 0.12t^3 - 2.13t^2 + 12.13t + 3.91$ , with  $t = 0$  corresponding to 9 a.m. and  $t = 12$  corresponding to 9 p.m. Compute the average value of  $b(t)$  to estimate the average number of hamburgers sold per hour.

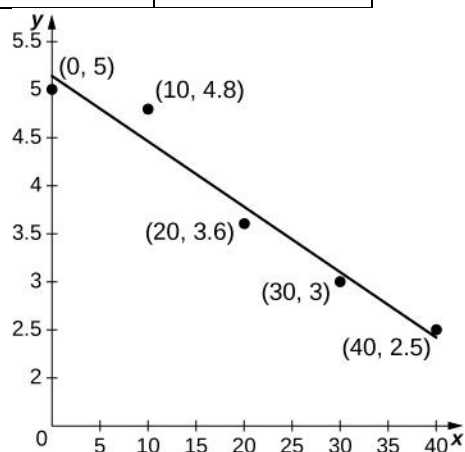
Hours Past Midnight	No. of Burgers Sold
9	3
12	28
15	20
18	30
21	45



Answer:  $\frac{1}{12} \int_0^{12} b(t) dt = \frac{0.12(12)^3}{4} - \frac{2.13(12)^2}{3} + \frac{12.13(12)}{2} + 3.91 = 26.29$  hamburgers per hour.

253. [T] An athlete runs by a motion detector, which records her speed, as displayed in the following table. The best linear fit to this data,  $\ell(t) = -0.068t + 5.14$ , is shown in the accompanying graph. Use the average value of  $\ell(t)$  between  $t = 0$  and  $t = 40$  to estimate the runner's average speed.

Minutes	Speed (m/sec)
0	5
10	4.8
20	3.6
30	3.0
40	2.5



Answer:  $\frac{1}{40} \int_0^{40} (-0.068t + 5.14) dt = -\frac{0.068(40)}{2} + 5.14 = 3.78 \text{ m/sec}$