

Chapter 4
Applications of Derivatives
4.3 Maxima and Minima

Section Exercises

90. In precalculus, you learned a formula for the position of the maximum or minimum of a quadratic equation $y = ax^2 + bx + c$, which was $h = -\frac{b}{(2a)}$. Prove this formula using calculus.

91. If you are finding an absolute minimum over an interval $[a, b]$, why do you need to check the endpoints? Draw a graph that supports your hypothesis.

Answer: Answers may vary

92. If you are examining a function over an interval (a, b) , for a and b finite, is it possible not to have an absolute maximum or absolute minimum?

Answer: Yes, one example would be $y = x$

93. When you are checking for critical points, explain why you also need to determine points where $f'(x)$ is undefined. Draw a graph to support your explanation.

Answer: Answers will vary

94. Can you have a finite absolute maximum for $y = ax^2 + bx + c$ over $(-\infty, \infty)$? Explain why or why not using graphical arguments.

Answer: Yes, if $a < 0$; answers will vary

95. Can you have a finite absolute maximum for $y = ax^3 + bx^2 + cx + d$ over $(-\infty, \infty)$ assuming a is non-zero? Explain why or why not using graphical arguments.

Answer: No; answers will vary

96. Let m be the number of local minima and M be the number of local maxima. Can you create a function where $M > m + 2$? Draw a graph to support your explanation.

Answer: Yes, only if the function is discontinuous; answers will vary

97. Is it possible to have more than one absolute maximum? Use a graphical argument to prove your hypothesis.

Answer: Since the absolute maximum is the function (output) value rather than the x value, the answer is no; answers will vary

98. Is it possible to have no absolute minimum or maximum for a function? If so, construct such a function. If not, explain why this is not possible.

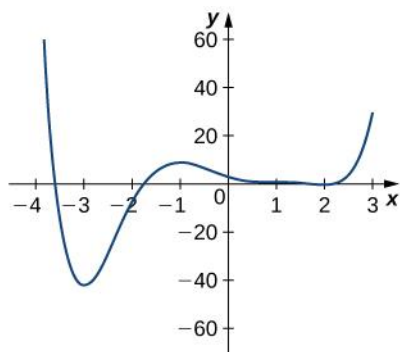
Answer: Yes; answers will vary

99. [T] Graph the function $y = e^{ax}$. For which values of a , on any infinite domain, will you have an absolute minimum and absolute maximum?

Answer: When $a = 0$

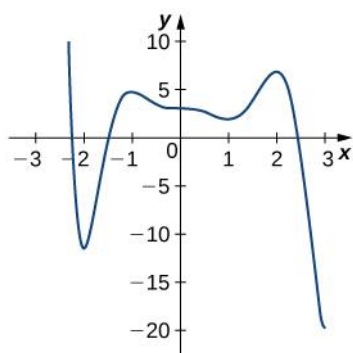
For the following exercises, determine where the local and absolute maxima and minima occur on the graph given. Assume the graph represents the entirety of each function.

100.



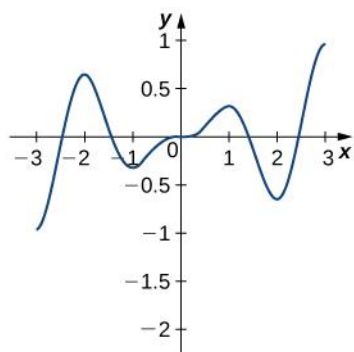
Answer: Absolute minimum at -3 ; absolute maximum at -4 ; local minimum at 2 ; local maximum at -1

101.



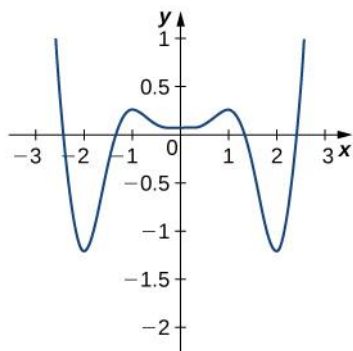
Answer: Absolute minimum at 3 ; Absolute maximum at -2.2 ; local minima at $-2, 1$; local maxima at $-1, 2$

102.



Answer: Absolute minimum at -3 ; absolute maximum at 3 ; local minima at $-1, 2$; local maxima at $-2, 1$

103.



Answer: Absolute minima at $-2, 2$; absolute maxima at $-2.5, 2.5$; local minimum at 0 ; local maxima at $-1, 1$

For the following problems, draw graphs of $f(x)$, which is continuous, over the interval $[-4, 4]$ with the following properties:

104. Absolute maximum at $x = 2$ and absolute minima at $x = \pm 3$

Answer: Answers may vary.

105. Absolute minimum at $x = 1$ and absolute maximum at $x = 2$

Answer: Answers may vary.

106. Absolute maximum at $x = 4$, absolute minimum at $x = -1$, local maximum at $x = -2$, and a critical point that is not a maximum or minimum at $x = 2$

Answer: Answers may vary.

107. Absolute maxima at $x = 2$ and $x = -3$, local minimum at $x = 1$, and absolute minimum at $x = 4$

Answer: Answers may vary.

For the following exercises, find the critical points in the domains of the following functions.

108. $y = 4x^3 - 3x$

Answer: $x = \pm \frac{1}{2}$

109. $y = 4\sqrt{x} - x^2$

Answer: $x = 1$

110. $y = \frac{1}{x-1}$

Answer: None

111. $y = \ln(x-2)$

Answer: None

112. $y = \tan(x)$

Answer: None

113. $y = \sqrt{4-x^2}$

Answer: $x = 0$

114. $y = x^{3/2} - 3x^{5/2}$

Answer: $x = 0, \frac{1}{5}$

115. $y = \frac{x^2 - 1}{x^2 + 2x - 3}$

Answer: None

116. $y = \sin^2(x)$

Answer: $n\pi, n\pi + \frac{\pi}{2}$

117. $y = x + \frac{1}{x}$

Answer: $x = -1, 1$

For the following exercises, find the local and/or absolute maxima for the functions over the specified domain.

118. $f(x) = x^2 + 3$ over $[-1, 4]$

Answer: Absolute maximum: $x = 4$, $f(x) = 19$; absolute minimum: $x = 0$, $f(x) = 3$

119. $y = x^2 + \frac{2}{x}$ over $[1, 4]$

Answer: Absolute maximum: $x = 4$, $y = \frac{33}{2}$; absolute minimum: $x = 1$, $y = 3$

120. $y = (x - x^2)^2$ over $[-1, 1]$

Answer: Absolute maximum: $x = -1$, $y = 4$; local maximum: $x = \frac{1}{2}$, $y = \frac{1}{16}$; absolute minima: $x = 0, 1$, $y = 0$

121. $y = \frac{1}{(x - x^2)}$ over $(0, 1)$

Answer: Absolute minimum: $x = \frac{1}{2}$, $y = 4$

122. $y = \sqrt{9 - x}$ over $[1, 9]$

Answer: Absolute maximum: $x = 1$, $y = \sqrt{8}$; absolute minima: $x = 9$, $y = 0$

123. $y = x + \sin(x)$ over $[0, 2\pi]$

Answer: Absolute maximum: $x = 2\pi$, $y = 2\pi$; absolute minimum: $x = 0$, $y = 0$

124. $y = \frac{x}{1 + x}$ over $[0, 100]$

Answer: Absolute maximum: $x = 100$, $y = \frac{100}{101}$; absolute minimum: $x = 0$, $y = 0$

125. $y = |x + 1| + |x - 1|$ over $[-3, 2]$

Answer: Absolute maximum: $x = -3$; absolute minimum: $-1 \leq x \leq 1$, $y = 2$

126. $y = \sqrt{x} - \sqrt{x^3}$ over $[0, 4]$

Answer: Absolute maximum: $x = \frac{1}{3}$, $y = \frac{2}{3\sqrt{3}}$; absolute minimum: $x = 4$, $y = -6$

127. $y = \sin x + \cos x$ over $[0, 2\pi]$

Answer: Absolute maximum: $x = \frac{\pi}{4}$, $y = \sqrt{2}$; absolute minimum: $x = \frac{5\pi}{4}$, $y = -\sqrt{2}$

128. $y = 4\sin q - 3\cos q$ over $[0, 2\pi]$

Answer: Absolute maximum: $\theta = \pi + \tan^{-1}\left(-\frac{4}{3}\right)$, $y = 5$; absolute minimum:

$$\theta = 2\pi + \tan^{-1}\left(-\frac{4}{3}\right), y = -5$$

For the following exercises, find the local and absolute minima and maxima for the functions over $(-\infty, \infty)$.

129. $y = x^2 + 4x + 5$

Answer: Absolute minimum: $x = -2$, $y = 1$

130. $y = x^3 - 12x$

Answer: Local maximum: $x = -2$, $y = 16$; local minimum: $x = 2$, $y = -16$

131. $y = 3x^4 + 8x^3 - 18x^2$

Answer: Absolute minimum: $x = -3$, $y = -135$; local maximum: $x = 0$, $y = 0$; local minimum: $x = 1$, $y = -7$

132. $y = x^3(1-x)^6$

Answer: Local maximum: $x = \frac{1}{3}$, $y = \frac{64}{19,683}$; local minimum: $x = 1$, $y = 0$

133. $y = \frac{x^2 + x + 6}{x - 1}$

Answer: Local maximum: $x = 1 - 2\sqrt{2}$, $y = 3 - 4\sqrt{2}$; local minimum: $x = 1 + 2\sqrt{2}$, $y = 3 + 4\sqrt{2}$

134. $y = \frac{x^2 - 1}{x - 1}$

Answer: None

For the following functions, use a calculator to graph the function and to estimate the absolute and local maxima and minima. Then, solve for them explicitly.

135. [T] $y = 3x\sqrt{1-x^2}$

Answer: Absolute maximum: $x = \frac{\sqrt{2}}{2}$, $y = \frac{3}{2}$; absolute minimum: $x = -\frac{\sqrt{2}}{2}$, $y = -\frac{3}{2}$

136. [T] $y = x + \sin(x)$

Answer: None

137. [T] $y = 12x^5 + 45x^4 + 20x^3 - 90x^2 - 120x + 3$

Answer: Local maximum: $x = -2$, $y = 59$; local minimum: $x = 1$, $y = -130$

138. [T] $y = \frac{x^3 + 6x^2 - x - 30}{x - 2}$

Answer: Absolute minimum: $x = -4$, $y = -1$

139. [T] $y = \frac{\sqrt{4-x^2}}{\sqrt{4+x^2}}$

Answer: Absolute maximum: $x = 0$, $y = 1$; absolute minimum: $x = -2$, 2 , $y = 0$

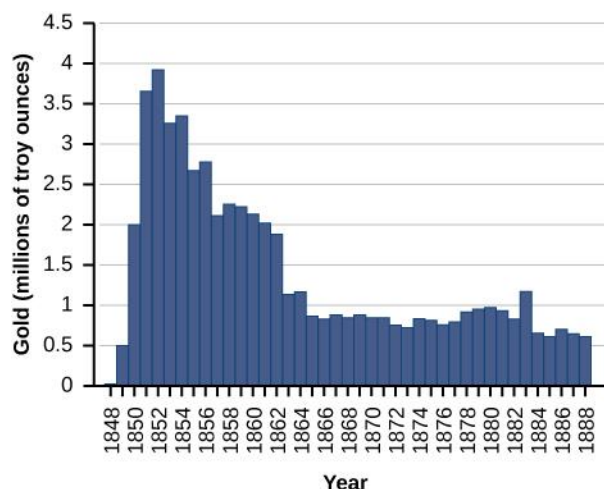
140. A company that produces cell phones has a cost function of $C = x^2 - 1200x + 36,400$, where C is cost in dollars and x is number of cell phones produced (in thousands). How many units of cell phone (in thousands) minimizes this cost function?

Answer: 600,000 cell phones

141. A ball is thrown into the air and its position is given by $h(t) = -4.9t^2 + 60t + 5$ m. Find the height at which the ball stops ascending. How long after it is thrown does this happen?

Answer: $h = \frac{9245}{49}$ m, $t = \frac{300}{49}$ s

For the following exercises, consider the production of gold during the California gold rush (1848–1888). The production of gold can be modeled by $G(t) = \frac{(25t)}{(t^2 + 16)}$, where t is the number of years since the rush began ($0 \leq t \leq 40$) and G is ounces of gold produced (in millions). A summary of the data is shown in the following figure.



142. Find when the maximum (local and global) gold production occurred, and the amount of gold produced during that maximum.

Answer: The global maximum occurred in 1852, when 3.9 million oz were produced; model predicted 3.125 million oz.

143. Find when the minimum (local and global) gold production occurred. What was the amount of gold produced during this minimum?

Answer: The global minimum was in 1848, when no gold was produced.

Find the critical points, maxima, and minima for the following piecewise functions.

144.
$$y = \begin{cases} x^2 - 4x & 0 \leq x \leq 1 \\ x^2 - 4 & 1 < x \leq 2 \end{cases}$$

Answer: Absolute minimum at $x = 1$, $y = -3$; absolute maxima at $x = 0$, $x = 2$, $y = 0$

145.
$$y = \begin{cases} x^2 + 1 & x \leq 1 \\ x^2 - 4x + 5 & x > 1 \end{cases}$$

Answer: Absolute minima: $x = 0$, $x = 2$, $y = 1$; local maximum at $x = 1$, $y = 2$

For the following exercises, find the critical points of the following generic functions. Are they maxima, minima, or neither? State the necessary conditions.

146. $y = ax^2 + bx + c$, given that $a > 0$

Answer: Minimum at $x = -\frac{b}{2a}$

147. $y = (x-1)^a$, given that $a > 1$ and a is an integer.”

Answer: No maxima/minima if a is odd, minimum at $x = 1$ if a is even

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