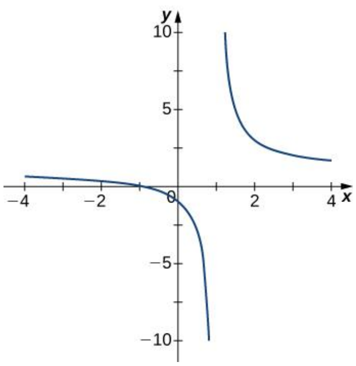
Chapter 4
Applications of Derivatives
4.6 Limits at Infinity and Asymptotes

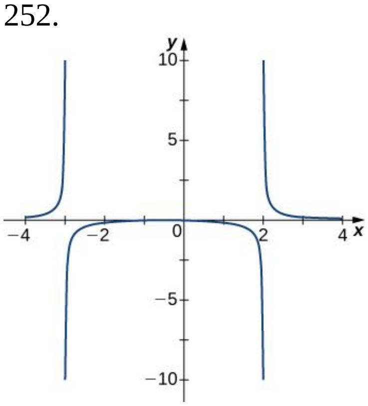
Section Exercises
For the following exercises, examine the graphs. Identify where the vertical asymptotes are located.

251.



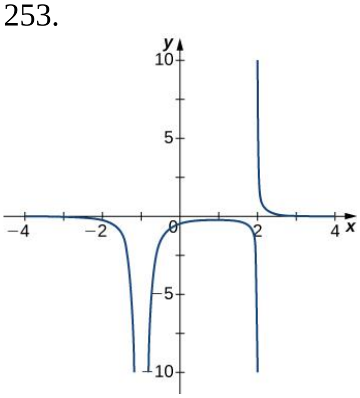
Answer: $x = 1$

252.



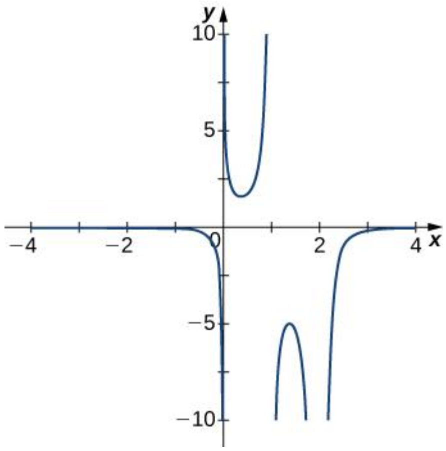
Answer: $x = 2, x = -3$

253.



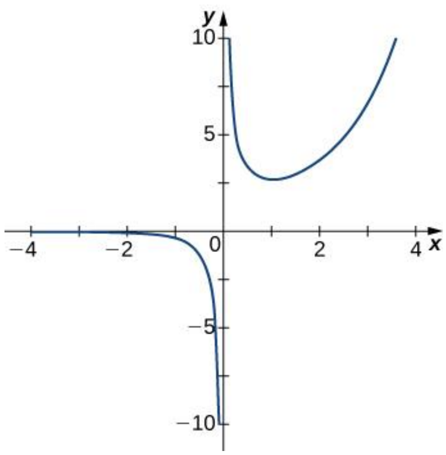
Answer: $x = -1, x = 2$

254.



Answer: $x = 0, x = 1, x = 2$

255.



Answer: $x = 0$

For the following functions $f(x)$, determine whether there is an asymptote at $x = a$. Justify your answer without graphing on a calculator.

256. $f(x) = \dfrac{x+1}{x^2+5x+4}, a = -1$

Answer: No, there is a removable discontinuity

257. $f(x) = \dfrac{x}{x-2}, a = 2$

Answer: Yes, there is a vertical asymptote

258. $f(x) = (x+2)^{3/2}, a = -2$

Answer: No

259. $f(x) = (x-1)^{-1/3}, a = 1$

Answer: Yes, there is vertical asymptote

260. $f(x) = 1 + x^{-2/5}, a = 1$

Answer: No, though there is one at $x = 0$

For the following exercises, evaluate the limit.

261. $\lim\limits_{x\to\infty} \dfrac{1}{3x+6}$

Answer: 0

262. $\lim\limits_{x\to\infty} \dfrac{2x-5}{4x}$

Answer: $\dfrac{1}{2}$

263. $\lim\limits_{x\to\infty} \dfrac{x^2-2x+5}{x+2}$

Answer: ∞

264. $\lim\limits_{x\to-\infty} \dfrac{3x^3-2x}{x^2+2x+8}$

Answer: $-\infty$

265. $\lim\limits_{x\to-\infty} \dfrac{x^4-4x^3+1}{2-2x^2-7x^4}$

Answer: $-\dfrac{1}{7}$

266. $\lim\limits_{x\to\infty} \dfrac{3x}{\sqrt{x^2+1}}$

Answer: 3

267. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{4x^2-1}}{x+2}$

Answer: -2

268. $\lim\limits_{x\to\infty} \dfrac{4x}{\sqrt{x^2-1}}$

Answer: 4

269. $\lim\limits_{x\to-\infty}\dfrac{4x}{\sqrt{x^2-1}}$

Answer: -4

270. $\lim\limits_{x\to\infty}\dfrac{2\sqrt{x}}{x-\sqrt{x}+1}$

Answer: 0

For the following exercises, find the horizontal and vertical asymptotes.

271. $f(x)=x-\dfrac{9}{x}$

Answer: Horizontal: none, vertical: $x=0$

272. $f(x)=\dfrac{1}{1-x^2}$

Answer: Horizontal: $y=0$, vertical: $x=\pm1$

273. $f(x)=\dfrac{x^3}{4-x^2}$

Answer: Horizontal: none, vertical: $x=\pm2$

274. $f(x)=\dfrac{x^2+3}{x^2+1}$

Answer: Horizontal: $y=1$, vertical: none

275. $f(x)=\sin(x)\sin(2x)$

Answer: Horizontal: none, vertical: none

276. $f(x)=\cos x+\cos(3x)+\cos(5x)$

Answer: Horizontal: none, vertical: none

277. $f(x)=\dfrac{x\sin(x)}{x^2-1}$

Answer: Horizontal: $y=0$, vertical: $x=\pm1$

278. $f(x)=\dfrac{x}{\sin(x)}$

Answer: Horizontal: none, vertical: $x=\pm\pi n$ for all $n\neq0$

279. $f(x) = \frac{1}{x^3 + x^2}$

Answer: Horizontal: $y = 0$, vertical: $x = 0$ and $x = -1$

280. $f(x) = \frac{1}{x-1} - 2x$

Answer: Horizontal: none, vertical: $x = 1$

281. $f(x) = \frac{x^3 + 1}{x^3 - 1}$

Answer: Horizontal: $y = 1$, vertical: $x = 1$

282. $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$

Answer: Horizontal: none, vertical: $x = \frac{\pi}{4} \pm n\pi$ for all n

283. $f(x) = x - \sin x$

Answer: Horizontal: none, vertical: none

284. $f(x) = \frac{1}{x} - \sqrt{x}$

Answer: Horizontal: none, vertical: $x = 0$

For the following exercises, construct a function $f(x)$ that has the given asymptotes.

285. $x = 1$ and $y = 2$

Answer: Answers will vary, for example: $y = \frac{2x}{x-1}$

286. $x = 1$ and $y = 0$

Answer: Answers will vary, for example: $y = \frac{1}{(x-1)}$

287. $y = 4$, $x = -1$

Answer: Answers will vary, for example: $y = \frac{4x}{x+1}$

288. $x = 0$

Answer: Answers will vary, for example: $y = \frac{1}{x}$

For the following exercises, graph the function on a graphing calculator on the window $x = [-5, 5]$ and estimate the horizontal asymptote or limit. Then, calculate the actual horizontal asymptote or limit.

289. **[T]** $f(x) = \frac{1}{x+10}$

Answer: $y = 0$

290. **[T]** $f(x) = \frac{x+1}{x^2+7x+6}$

Answer: $y = 0$

291. **[T]** $\lim_{x \to -\infty} x^2 + 10x + 25$

Answer: ∞

292. **[T]** $\lim_{x \to -\infty} \frac{x+2}{x^2+7x+6}$

Answer: $y = 0$

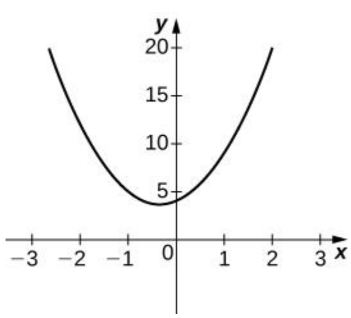
293. **[T]** $\lim_{x \to \infty} \frac{3x+2}{x+5}$

Answer: $y = 3$

For the following exercises, draw a graph of the functions without using a calculator. Be sure to notice all important features of the graph: local maxima and minima, inflection points, and asymptotic behavior.

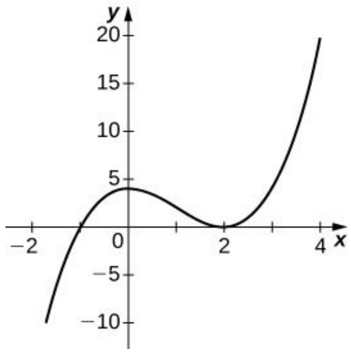
294. $y = 3x^2 + 2x + 4$

Answer:



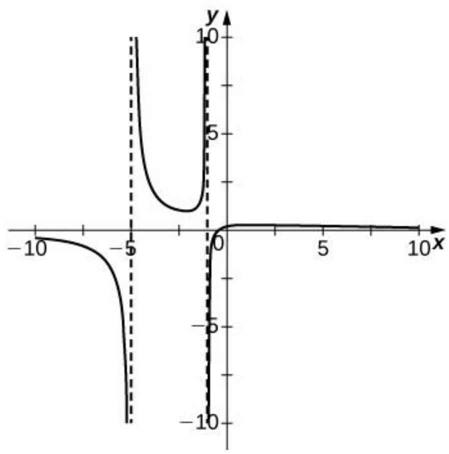
295. $y = x^3 - 3x^2 + 4$

Answer:



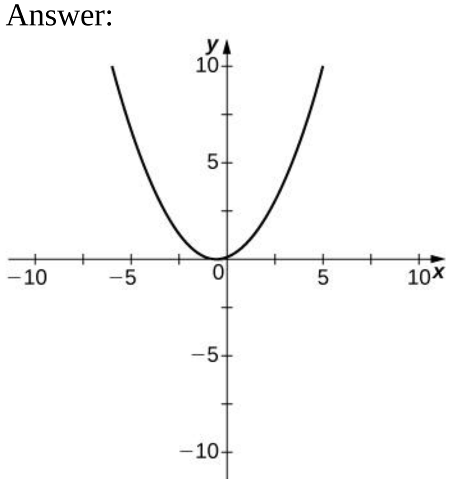
296. $y = \dfrac{2x+1}{x^2+6x+5}$

Answer:



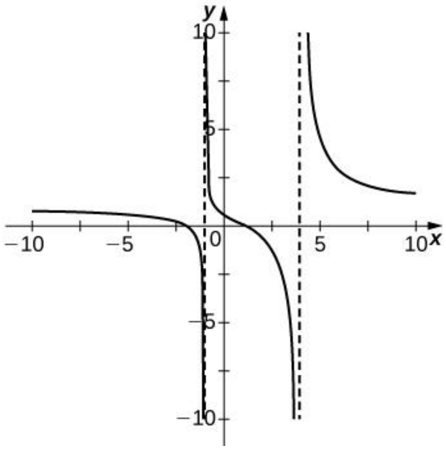
297. $y = \dfrac{x^3+4x^2+3x}{3x+9}$

Answer:



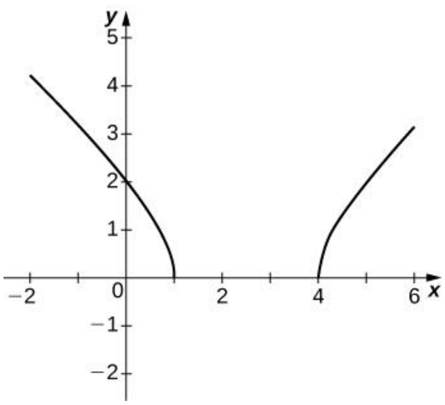
298. $y = \dfrac{x^2 + x - 2}{x^2 - 3x - 4}$

Answer:



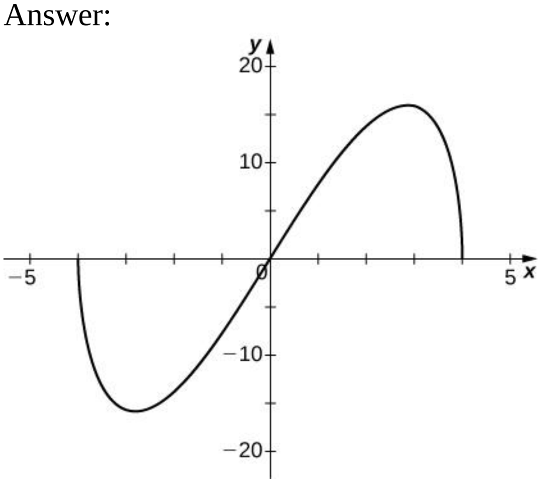
299. $y = \sqrt{x^2 - 5x + 4}$

Answer:



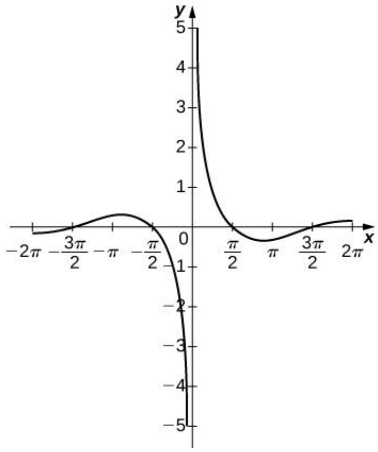
300. $y = 2x\sqrt{16 - x^2}$

Answer:



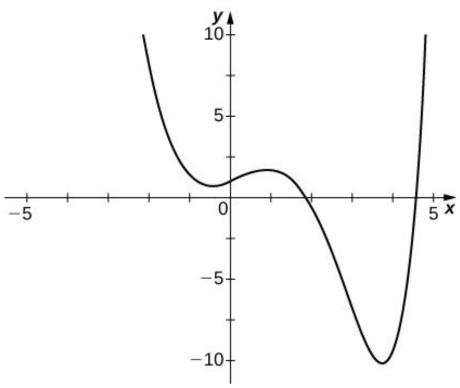
301. $y = \dfrac{\cos x}{x}$, on $x = [-2\pi, 2\pi]$

Answer:



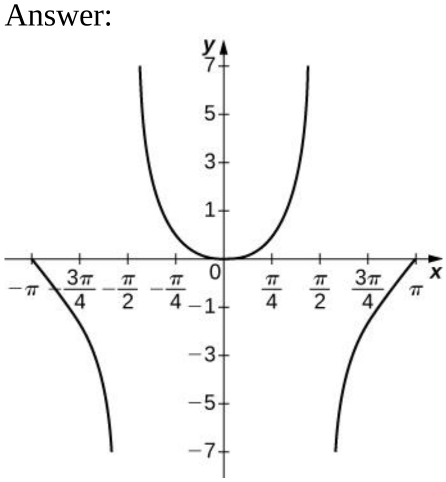
302. $y = e^x - x^3$

Answer:



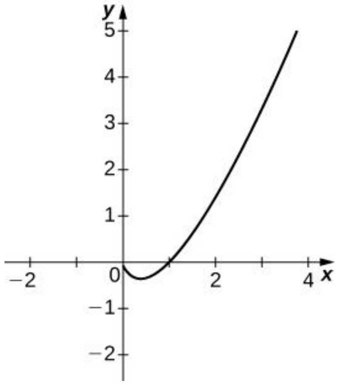
303. $y = x\tan x, x = [-\pi, \pi]$

Answer:



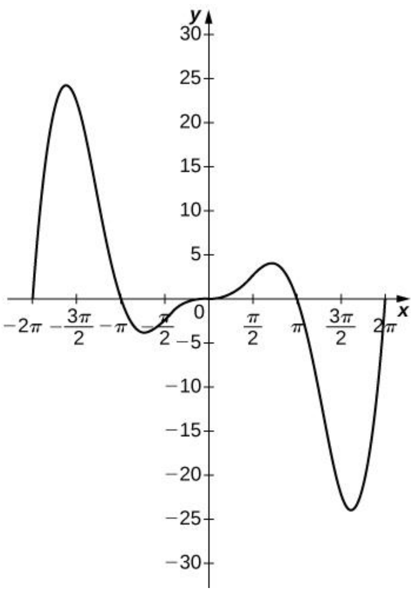
304. $y = x\ln(x), x > 0$

Answer:



305. $y = x^2 \sin(x), x = [-2\pi, 2\pi]$

Answer:



306. For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $y = 2$ then the polynomials $P(x)$ and $Q(x)$ must have what relation?

Answer: The leading term of $P(x)$ is twice the leading term of $Q(x)$. And the leading terms of $P(x)$ and $Q(x)$. have the same degree.

307. For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $x = 0$, then the polynomials $P(x)$ and $Q(x)$. must have what relation?

Answer: $Q(x)$. must have x^{k+1} as a factor, where $P(x)$ has x^k as a factor.

308. If $f'(x)$ has asymptotes at $y=3$ and $x=1$, then $f(x)$ has what asymptotes?
Answer: Nothing can be said about the asymptotes of $f(x)$

309. Both $f(x)=\frac{1}{(x-1)}$ and $g(x)=\frac{1}{(x-1)^2}$ have asymptotes at $x=1$ and $y=0$. What is the most obvious difference between these two functions?
Answer: $\lim\limits_{x\to 1^-} f(x)$ and $\lim\limits_{x\to 1^-} g(x)$

310. True or false: Every ratio of polynomials has vertical asymptotes.
Answer: False