

Chapter 4
Applications of Derivatives
4.8 L'Hôpital's Rule

Section Exercises

For the following exercises, evaluate the limit.

356. Evaluate the limit $\lim_{x \rightarrow \infty} \frac{e^x}{x}$.

Answer: ∞

357. Evaluate the limit $\lim_{x \rightarrow \infty} \frac{e^x}{x^k}$.

Answer: ∞

358. Evaluate the limit $\lim_{x \rightarrow \infty} \frac{\ln x}{x^k}$.

Answer: 0

359. Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^2 - a^2}$, $a \neq 0$.

Answer: $\frac{1}{2a}$

360. Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^3 - a^3}$, $a \neq 0$.

Answer: $\frac{1}{3a^2}$

361. Evaluate the limit $\lim_{x \rightarrow a} \frac{x - a}{x^n - a^n}$, $a \neq 0$.

Answer: $\frac{1}{na^{n-1}}$

For the following exercises, determine whether you can apply L'Hôpital's rule directly. Explain why or why not. Then, indicate if there is some way you can alter the limit so you can apply L'Hôpital's rule.

362. $\lim_{x \rightarrow 0^+} x^2 \ln x$

Answer: Cannot apply directly; write as $\lim_{x \rightarrow 0^+} \frac{\ln x}{1/x^2}$ to get indeterminate form

363. $\lim_{x \rightarrow \infty} x^{1/x}$

Answer: Cannot apply directly; use logarithms

364. $\lim_{x \rightarrow 0} x^{2/x}$

Answer: Cannot apply directly; use logarithms

365. $\lim_{x \rightarrow 0} \frac{x^2}{1/x}$

Answer: Cannot apply directly; rewrite as $\lim_{x \rightarrow 0} x^3$

366. $\lim_{x \rightarrow \infty} \frac{e^x}{x}$

Answer: Can apply directly

For the following exercises, evaluate the limits with either L'Hôpital's rule or previously learned methods.

367. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

Answer: 6

368. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3}$

Answer: 0

369. $\lim_{x \rightarrow 0} \frac{(1+x)^{-2} - 1}{x}$

Answer: -2

370. $\lim_{x \rightarrow \pi/2} \frac{\cos x}{(\pi/2) - x}$

Answer: 1

371. $\lim_{x \rightarrow \pi} \frac{x - \pi}{\sin x}$

Answer: -1

372. $\lim_{x \rightarrow 1} \frac{x - 1}{\sin x}$

Answer: 0

$$373. \quad \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x}$$

Answer: n

$$374. \quad \lim_{x \rightarrow 0} \frac{(1+x)^n - 1 - nx}{x^2}$$

Answer: $\frac{n}{2}(n-1)$

$$375. \quad \lim_{x \rightarrow 0} \frac{\sin x - \tan x}{x^3}$$

Answer: $-\frac{1}{2}$

$$376. \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

Answer: 1

$$377. \quad \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$$

Answer: $\frac{1}{2}$

$$378. \quad \lim_{x \rightarrow 0} \frac{\tan x}{\sqrt{x}}$$

Answer: 0

$$379. \quad \lim_{x \rightarrow 1} \frac{x-1}{\ln x}$$

Answer: 1

$$380. \quad \lim_{x \rightarrow 0} (x+1)^{1/x}$$

Answer: e

$$381. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x} - \sqrt[3]{x}}{x-1}$$

Answer: $\frac{1}{6}$

$$382. \quad \lim_{x \rightarrow 0^+} x^{2x}$$

Answer: 1

$$383. \quad \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$$

Answer: 1

$$384. \quad \lim_{x \rightarrow 0} \frac{\sin x - x}{x^2}$$

Answer: 0

$$385. \quad \lim_{x \rightarrow 0^+} x \ln(x^4)$$

Answer: 0

$$386. \quad \lim_{x \rightarrow \infty} (x - e^x)$$

Answer: $-\infty$

$$387. \quad \lim_{x \rightarrow \infty} x^2 e^{-x}$$

Answer: 0

$$388. \quad \lim_{x \rightarrow 0} \frac{3^x - 2^x}{x}$$

Answer: $\ln(3/2)$

$$389. \quad \lim_{x \rightarrow 0} \frac{1 + 1/x}{1 - 1/x}$$

Answer: -1

$$390. \quad \lim_{x \rightarrow \pi/4} (1 - \tan x) \cot x$$

Answer: 0

$$391. \quad \lim_{x \rightarrow \infty} x e^{1/x}$$

Answer: ∞

$$392. \quad \lim_{x \rightarrow 0^+} x^{1/\cos x}$$

Answer: 0

$$393. \quad \lim_{x \rightarrow 0^+} x^{1/x}$$

Answer: 0

$$394. \quad \lim_{x \rightarrow 0^-} \left(1 - \frac{1}{x}\right)^x$$

Answer: 1

$$395. \quad \lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$$

Answer: $\frac{1}{e}$

For the following exercises, use a calculator to graph the function and estimate the value of the limit, then use L'Hôpital's rule to find the limit directly.

$$396. \quad [\mathbf{T}] \lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

Answer: 1

$$397. \quad [\mathbf{T}] \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$$

Answer: 0

$$398. \quad [\mathbf{T}] \lim_{x \rightarrow 1} \frac{x - 1}{1 - \cos(\pi x)}$$

Answer: 0

$$399. \quad [\mathbf{T}] \lim_{x \rightarrow 1} \frac{e^{(x-1)} - 1}{x - 1}$$

Answer: 1

$$400. \quad [\mathbf{T}] \lim_{x \rightarrow 1} \frac{(x-1)^2}{\ln x}$$

Answer: 0

$$401. \quad [\mathbf{T}] \lim_{x \rightarrow \pi} \frac{1 + \cos x}{\sin x}$$

Answer: 0

$$402. \quad [\mathbf{T}] \lim_{x \rightarrow 0} \left(\csc x - \frac{1}{x} \right)$$

Answer: 0

403. [T] $\lim_{x \rightarrow 0^+} \tan(x^x)$

Answer: $\tan(1)$

404. [T] $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sin x}$

Answer: $-\infty$

405. [T] $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$

Answer: 2

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