Chapter 3
Derivatives
3.1 Defining the Derivative

Section Exercises

For the following exercises, use Equation_03_01_03 to find the slope of the secant line between the values x_1 and x_2 for each function $y = f(x)$.

1. $f(x) = 4x + 7; x_1 = 2, x_2 = 5$

Answer: 4

2. $f(x) = 8x - 3; x_1 = -1, x_2 = 3$

Answer: 8

3. $f(x) = x^2 + 2x + 1; x_1 = 3, x_2 = 3.5$

Answer: 8.5

4. $f(x) = -x^2 + x + 2; x_1 = 0.5, x_2 = 1.5$

Answer: -1

5. $f(x) = \dfrac{4}{3x-1}; x_1 = 1, x_2 = 3$

Answer: $-\dfrac{3}{4}$

6. $f(x) = \dfrac{x-7}{2x+1}; x_1 = -2, x_2 = 0$

Answer: 3

7. $f(x) = \sqrt{x}; x_1 = 1, x_2 = 16$

Answer: 0.2

8. $f(x) = \sqrt{x-9}; x_1 = 10, x_2 = 13$

Answer: $\dfrac{1}{3}$

9. $f(x) = x^{1/3} + 1; x_1 = 0, x_2 = 8$

Answer: 0.25

10. $f(x) = 6x^{2/3} + 2x^{1/3}; x_1 = 1, x_2 = 27$

Answer: 2

For the following functions,

a. use Equation_03_01_04 to find the slope of the tangent line $m_{\tan} = f'(a)$, and

b. find the equation of the tangent line to f at $x = a$.

11. $f(x) = 3 - 4x,\ a = 2$

Answer: a. -4 b. $y = 3 - 4x$

12. $f(x) = \frac{x}{5} + 6,\ a = -1$

Answer: a. $\frac{1}{5}$ b. $y = \frac{x}{5} + 6$

13. $f(x) = x^2 + x,\ a = 1$

Answer: a. 3 b. $y = 3x - 1$

14. $f(x) = 1 - x - x^2,\ a = 0$

Answer: a. -1 b. $y = -x + 1$

15. $f(x) = \frac{7}{x},\ a = 3$

Answer: a. $\frac{-7}{9}$ b. $y = \frac{-7}{9}x + \frac{14}{3}$

16. $f(x) = \sqrt{x+8},\ a = 1$

Answer: a. $\frac{1}{6}$ b. $y = \frac{1}{6}x + \frac{17}{6}$

17. $f(x) = 2 - 3x^2, a = -2$

Answer: a. 12 b. $y = 12x + 14$

18. $f(x) = \frac{-3}{x-1}, a = 4$

Answer: a. $\frac{1}{3}$ b. $y = \frac{1}{3}x - \frac{7}{3}$

19. $f(x)=\dfrac{2}{x+3}, a=-4$

Answer: a. -2 b. $y=-2x-10$

20. $f(x)=\dfrac{3}{x^2}, a=3$]

Answer: a. $-\dfrac{2}{9}$ b. $y=-\dfrac{2}{9}x+1$

For the following functions $y=f(x)$, , find $f'(a)$ using Equation_03_01_03.

21. $f(x)=5x+4, a=-1$

Answer: 5

22. $f(x)=-7x+1, a=3$

Answer: -7

23. $f(x)=x^2+9x, a=2$

Answer: 13

24. $f(x)=3x^2-x+2, a=1$

Answer: 5

25. $f(x)=\sqrt{x}, a=4$

Answer: $\dfrac{1}{4}$

26. $f(x)=\sqrt{x-2}, a=6$

Answer: $\dfrac{1}{4}$

27. $f(x)=\dfrac{1}{x}, a=2$

Answer: $-\dfrac{1}{4}$

28. $f(x)=\dfrac{1}{x-3}, a=-1$

Answer: $-\dfrac{1}{16}$

29. $f(x)=\frac{1}{x^3}, a=1$

Answer: -3

30. $f(x)=\frac{1}{\sqrt{x}}, a=4$

Answer: $-\frac{1}{16}$

For the following exercises, given the function $y=f(x)$,

- **a. find the slope of the secant line PQ for each point $Q(x, f(x))$ with x value given in the table.**
- **b. Use the answers from a. to estimate the value of the slope of the tangent line at P.**
- **c. Use the answer from b. to find the equation of the tangent line to f at point P.**

31. **[T]** $f(x)=x^2+3x+4,\ P(1,8)$ (Round to 6 decimal places.)

x	Slope m_{PQ}	x	Slope m_{PQ}
1.1	(i)	0.9	(vii)
1.01	(ii)	0.99	(viii)
1.001	(iii)	0.999	(ix)
1.0001	(iv)	0.9999	(x)
1.00001	(v)	0.99999	(xi)
1.000001	(vi)	0.999999	(xii)

Answer: a. (i) 5.100000, (ii) 5.010000, (iii) 5.001000, (iv) 5.000100, (v) 5.000010, (vi) 5.000001, (vii) 4.900000, (viii) 4.990000, (ix) 4.999000, (x) 4.999900, (xi) 4.999990, (x) 4.999999 b. $m_{\tan}=5$ c. $y=5x+3$

32. **[T]** $f(x)=\frac{x+1}{x^2-1}, P(0,-1)$

x	Slope m_{PQ}	x	Slope m_{PQ}
0.1	(i)	–0.1	(vii)
0.01	(ii)	–0.01	(viii)
0.001	(iii)	–0.001	(ix)
0.0001	(iv)	–0.0001	(x)
0.00001	(v)	–0.00001	(xi)
0.000001	(vi)	–0.000001	(xii)

Answer: a. (i) -1.111111, (ii) -1.010101, (iii) -1.001001, (iv) -1.000100, (v) -1.000010, (vi) -1.000001, (vii) -0.909090, (viii) -0.990099, (ix) -0.999001, (x) -0.999900, (xi) -0.999990, (xii) -0.999999 b. $m_{\tan}=-1$ c. $y=-x-1$

33. **[T]** $f(x)=10e^{0.5x}, P(0,10)$ (Round to 4 decimal places.)

x	Slope m_{PQ}
−0.1	(i)
−0.01	(ii)
−0.001	(iii)
−0.0001	(iv)
−0.00001	(v)
−0.000001	(vi)

Answer: a. (i) 4.8771, (ii) 4.9875, (iii) 4.9988 , (iv) 4.9999, (v) 4.9999, (vi) 4.9999 b. $m_{\tan}=5$ c. $y=5x+10$

34. **[T]** $f(x)=\tan(x), P(\pi,0)$

x	Slope m_{PQ}
3.1	(i)
3.14	(ii)
3.141	(iii)
3.1415	(iv)
3.14159	(v)
3.141592	(vi)

Answer: a. (i) 1.0006, (ii) 1, (iii) 1, (iv) 1, (v) 1, (vi) 1 b. $m_{\tan}=1$ c. $y=x-\pi$

[T] For the following position functions $y=s(t)$, an object is moving along a straight line, where t is in seconds and s is in meters. Find

- **a. the simplified expression for the average velocity from $t=2$ to $t=2+h$;**
- **b. the average velocity between $t=2$ and $t=2+h$, where (i) $h=0.1$, (ii) $h=0.01$, (iii) $h=0.001$, and (iv) $h=0.0001$; and**
- **c. use the answer from a. to estimate the instantaneous velocity at $t=2$ second.**

35. $s(t)=\frac{1}{3}t+5$

Answer: a. $\frac{1}{3}$; b. (i) $0.\overline{3}$ m/s, (ii) $0.\overline{3}$ m/s, (iii) $0.\overline{3}$ m/s, (iv) $0.\overline{3}$ m/s; c. $0.\overline{3}=\frac{1}{3}$ m/s

36. $s(t)=t^2-2t$

Answer: a. $(h+2)$; b. (i) 2.1 m/s, (ii) 2.01 m/s, (iii) 2.001 m/s, (iv) 2.0001 m/s; c. 2 m/s

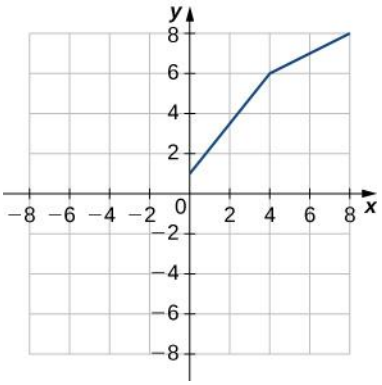
37. $s(t)=2t^3+3$

Answer: a. $2(h^2+6h+12)$; b. (i) 25.22 m/s, (ii) 24.12 m/s, (iii) 24.01 m/s, (iv) 24 m/s; c. 24 m/s

38. $s(t)=\dfrac{16}{t^2}-\dfrac{4}{t}$

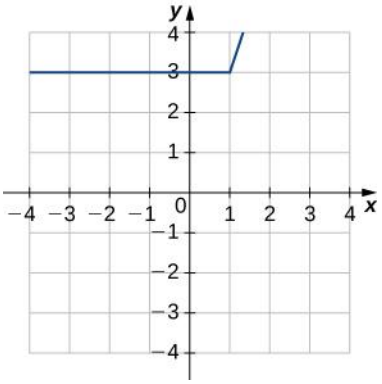
Answer: a. $-\dfrac{2(h+6)}{(h+2)^2}$; b. (i) -2.7664 m/s, (ii) -2.9752 m/s, (iii) -2.9975 m/s, (iv) -2.9998 m/s; c. -3 m/s

39. Use the following graph to evaluate a. $f'(1)$ and b. $f'(6)$.



Answer: a. 1.25; b. 0.5

40. Use the following graph to evaluate a. $f'(-3)$ and b. $f'(1.5)$.



Answer: a. 0; b. 2

For the following exercises, use the limit definition of derivative to show that the derivative does not exist at $x = a$ for each of the given functions.

41. $f(x)=x^{1/3}, x=0$

Answer: $\lim\limits_{x\to 0^-}\dfrac{x^{1/3}-0}{x-0}=\lim\limits_{x\to 0^-}\dfrac{1}{x^{2/3}}=\infty$

42. $f(x) = x^{2/3}, x = 0$

Answer: $\lim_{x \to 0^-} \frac{x^{2/3} - 0}{x - 0} = \lim_{x \to 0^-} \frac{1}{x^{1/3}} = -\infty$

43. $f(x) = \begin{cases} 1, x < 1 \\ x, x \geq 1 \end{cases}, x = 1$

Answer: $\lim_{x \to 1^-} \frac{1-1}{x-1} = 0 \neq 1 = \lim_{x \to 1^+} \frac{x-1}{x-1}$

44. $f(x) = \frac{|x|}{x}, x = 0$

Answer: $\lim_{x \to 0^-} \frac{|x|}{x} = -1 \neq 1 = \lim_{x \to 0^+} \frac{|x|}{x}$

45. **[T]** The position in feet of a race car along a straight track after t seconds is modeled by the function $s(t) = 8t^2 - \frac{1}{16}t^3$.

a. Find the average velocity of the vehicle over the following time intervals to four decimal places:
   (i) [4, 4.1]
   (ii) [4, 4.01]
   (iii)[4, 4.001]
   (iv)[4, 4.0001]

b. Use a. to draw a conclusion about the instantaneous velocity of the vehicle at $t = 4$ seconds.

Answer: a. (i) 61.7244 ft/s, (ii) 61.0725 ft/s (iii) 61.0072 ft/s (iv) 61.0007 ft/s b. At 4 seconds the race car is traveling at a rate/velocity of 61 ft/s

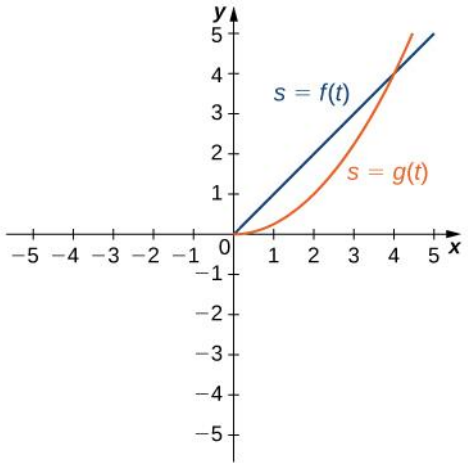
46. [**T]** The distance in feet that a ball rolls down an incline is modeled by the function $s(t) = 14t^2$, where t is seconds after the ball begins rolling.

a. Find the average velocity of the ball over the following time intervals:
   (i) [5, 5.1]
   (ii) [5, 5.01]
   (iii)[5, 5.001]
   (iv)[5, 5.0001]

b. Use the answers from a. to draw a conclusion about the instantaneous velocity of the ball at $t = 5$ seconds.

Answer: a. (i) 141 ft/s, (ii) 140.14 ft/s, (iii) 140.014 ft/s, (iv) 140.0014 ft/s b. The ball is traveling at a rate/velocity of 140 ft/s at $t = 5$ seconds.

47. Two vehicles start out traveling side by side along a straight road. Their position functions, shown in the following graph, are given by $s = f(t)$ and $s = g(t)$, where s is measured in feet and t is measured in seconds.



a. Which vehicle has traveled farther at $t = 2$ seconds?
b. What is the approximate velocity of each vehicle at $t = 3$ seconds?
c. Which vehicle is traveling faster at $t = 4$ seconds?
d. What is true about the positions of the vehicles at $t = 4$ seconds?

Answer: a. The vehicle represented by $f(t)$, because it has traveled 2 feet, whereas $g(t)$ has traveled 1 foot. b. The velocity of $f(t)$ is constant at 1 ft/s, while the velocity of $g(t)$ is approximately 2 ft/s. c. The vehicle represented by $g(t)$, with a velocity of approximately 4 ft/s. d. Both have traveled 4 feet in 4 seconds

48. [**T]** The total cost $C(x)$, in hundreds of dollars, to produce x jars of mayonnaise is given by

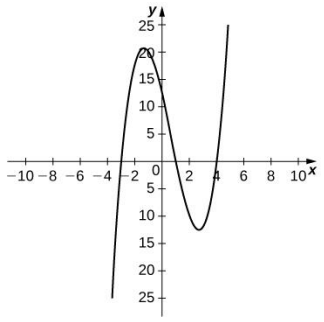
$C(x) = 0.000003x^3 + 4x + 300.$

a. Calculate the average cost per jar over the following intervals:
   (i) [100, 100.1]
   (ii) [100, 100.01]
   (iii)[100, 100.001]
   (iv)[100, 100.0001]
b. Use the answers from a. to estimate the average cost to produce 100 jars of mayonnaise.

Answer: a. (i) $4.09, (ii) $4.09, (iii) $4.09, (iv) $4.09 b. $4.09

49. **[T]** For the function $f(x) = x^3 - 2x^2 - 11x + 12,$ do the following.
   a. Use a graphing calculator to graph *f* in an appropriate viewing window.
   b. Use the ZOOM feature on the calculator to approximate the two values of $x = a$ for which $m_{\tan} = f'(a) = 0.$

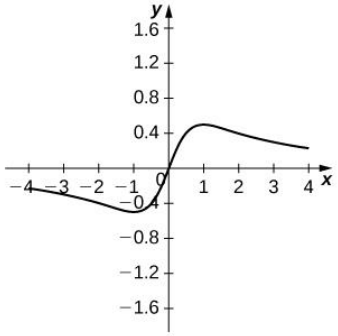
Answer: a.



b. $a \approx -1.361, 2.694$

50. [**T]** For the function $f(x) = \dfrac{x}{1+x^2},$ do the following.
   a. Use a graphing calculator to graph f in an appropriate viewing window.
   b. Use the ZOOM feature on the calculator to approximate the values of $x = a$ for which $m_{\tan} = f'(a) = 0.$

Answer: a.



b. $x = 1, -1$

51. Suppose that $N(x)$ computes the number of gallons of gas used by a vehicle traveling x miles. Suppose the vehicle gets 30 mpg.
   a. Find a mathematical expression for $N(x).$
   b. (b)What is $N(100)$? Explain the physical meaning.
   c. What is $N'(100)$? Explain the physical meaning.

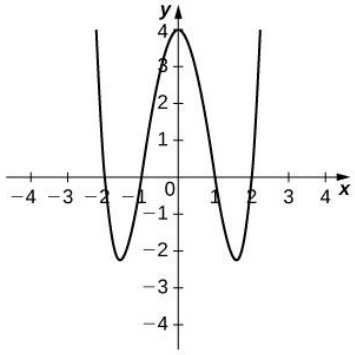
Answer: a. $N(x) = \dfrac{x}{30}$ b. 3.3 gallons. When the vehicle travels 100 miles, it has used 3.3 gallons of gas. c. $\dfrac{1}{30}.$ The rate of gas consumption in gallons per mile that the vehicle is achieving after having traveled 100 miles.

52. [**T]** For the function $f(x) = x^4 - 5x^2 + 4,$ do the following.

a. Use a graphing calculator to graph f in an appropriate viewing window.

b. Use the nDeriv function, which numerically finds the derivative, on a graphing calculator to estimate $f'(-2),\ f'(-0.5), f'(1.7),$ and $f'(2.718).$

Answer: a.



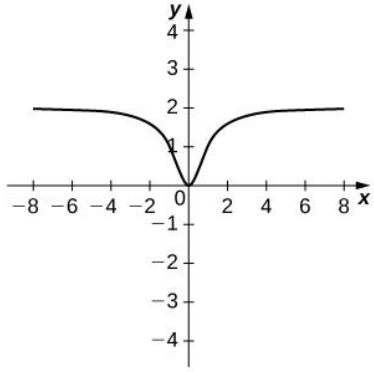
b. –12, 4.5, 2.65, 53.14

53. [**T]** For the function $f(x) = \dfrac{x^2}{x^2+1},$ do the following.

a. Use a graphing calculator to graph f in an appropriate viewing window.

b. Use the nDeriv function on a graphing calculator to find $f'(-4),\ f'(-2), f'(2),$ and $f'(4).$

Answer: a.



b. –0.028, –0.16, 0.16, 0.028