Chapter 3
Derivatives
3.5 Derivatives of Trigonometric Functions

Section Exercises

For the following exercises, find $\frac{dy}{dx}$ for the given functions.

175. $y = x^2 - \sec x + 1$

Answer: $\frac{dy}{dx} = 2x - \sec x \tan x$

176. $y = 3\csc x + \frac{5}{x}$

Answer: $\frac{dy}{dx} = -3\csc x \cot x - \frac{5}{x^2}$

177. $y = x^2 \cot x$

Answer: $\frac{dy}{dx} = 2x \cot x - x^2 \csc^2 x$

178. $y = x - x^3 \sin x$

Answer: $\frac{dy}{dx} = 1 - 3x^2 \sin x - x^3 \cos x$

179. $y = \frac{\sec x}{x}$

Answer: $\frac{dy}{dx} = \frac{x \sec x \tan x - \sec x}{x^2}$

180. $y = \sin x \tan x$

Answer: $\frac{dy}{dx} = \cos x \tan x + \sin x \sec^2 x$

181. $y = (x + \cos x)(1 - \sin x)$

Answer: $\frac{dy}{dx} = (1 - \sin x)(1 - \sin x) - \cos x (x + \cos x)$

182. $y=\dfrac{\tan x}{1-\sec x}$

Answer: $\dfrac{dy}{dx}=\dfrac{(1-\sec x)sec^2 x+\sec x tan^2 x}{(1-\sec x)^2}$

183. $y=\dfrac{1-\cot x}{1+\cot x}$

Answer: $\dfrac{dy}{dx}=\dfrac{2csc^2 x}{(1+\cot x)^2}$

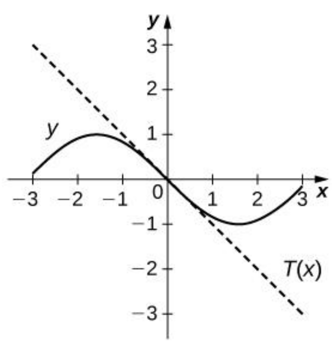
184. $y=\cos x(1+\csc x)$

Answer: $\dfrac{dy}{dx}=-\sin x(1+\csc x)-\cos x\csc x\cot x=-\sin x-$

For the following exercises, find the equation of the tangent line to each of the given functions at the indicated values of x. Then use a calculator to graph both the function and the tangent line to ensure the equation for the tangent line is correct.

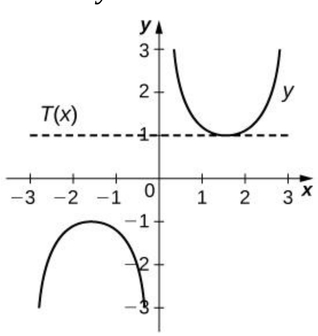
185. **[T]** $f(x)=-\sin x,\;\; x=0$

Answer: $y=-x$



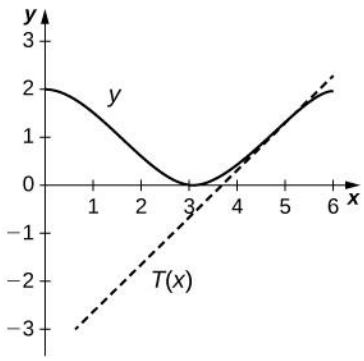
186. **[T]** $f(x)=\csc x,\; x=\dfrac{\pi}{2}$

Answer: $y=1$



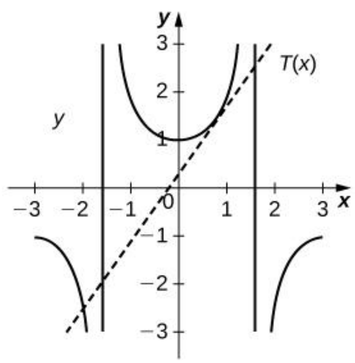
187. **[T]** $f\left(x\right)=1+\cos x,\ x=\frac{3\pi}{2}$

Answer: $y=x+\frac{2-3\pi}{2}$



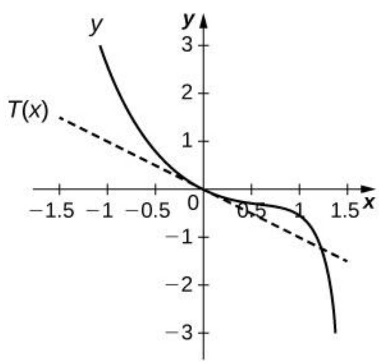
188. **[T]** $f\left(x\right)=\sec x,\ x=\frac{\pi}{4}$

Answer: $y=\sqrt{2}x+\frac{\sqrt{2}\left(4-\pi\right)}{4}$



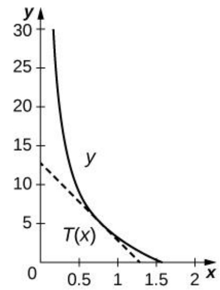
189. **[T]** $f\left(x\right)=x^2-\tan x,\ \ x=0$

Answer: $y=-x$



190. **[T]** $f(x) = 5\cot x,\ \ x = \frac{\pi}{4}$

Answer: $y = 5 + \frac{5\pi}{2} - 10x$



For the following exercises, find $\frac{d^2y}{dx^2}$ for the given functions.

191. $y = x\sin x - \cos x$

Answer: $3\cos x - x\sin x$

192. $y = \sin x\cos x$

Answer: $-4\sin x\cos x$

193. $y = x - \frac{1}{2}\sin x$

Answer: $\frac{1}{2}\sin x$

194. $y = \frac{1}{x} + \tan x$

Answer: $2\sec^2 x\tan x + \frac{2}{x^3}$

195. $y = 2\csc x$

Answer: $2\csc x\left(\csc^2 x + \cot^2 x\right)$

196. $y = \sec^2 x$

Answer: $2\sec^4 x + 4\sec^2 x\tan^2 x$

197. Find all x values on the graph of $f(x) = -3\sin x\cos x$ where the tangent line is horizontal.

Answer: $\frac{(2n+1)\pi}{4}$, where n is an integer

198. Find all x values on the graph of $f(x) = x - 2\cos x$ for $0 < x < 2\pi$ where the tangent line has slope 2.

Answer: $\frac{\pi}{6}, \frac{5\pi}{6}$

199. Let $f(x) = \cot x$. Determine the points on the graph of f for $0 < x < 2\pi$ where the tangent line(s) is (are) parallel to the line $y = -2x$.

Answer: $\left(\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, -1\right)$

200. **[T]** A mass on a spring bounces up and down in simple harmonic motion, modeled by the function $s(t) = -6\cos t$ where s is measured in inches and t is measured in seconds. Find the rate at which the spring is oscillating at $t = 5$ s.

Answer: $6\sin(5) \sim -5.754$ in./s

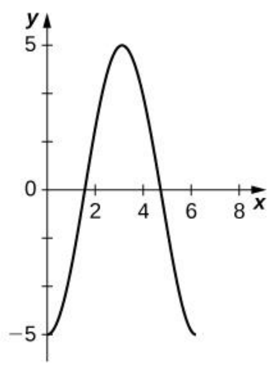
201. Let the position of a swinging pendulum in simple harmonic motion be given by $s(t) = a\cos t + b\sin t$ where a and b are constants, t measures time in seconds, and s measures position in centimeters. If the position is 0 cm and velocity is 3 cm/s when t = 0, find the values of a and b.

Answer: $a = 0, b = 3$

202. After a diver jumps off a diving board, the edge of the board oscillates with position given by $s(t) = -5\cos t$ cm at t seconds after the jump.

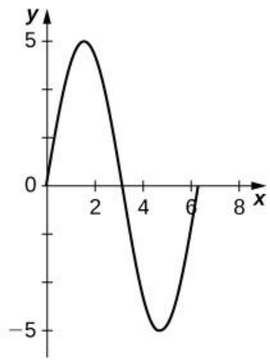
a. Sketch one period of the position function for $t \geq 0$.
b. Find the velocity function.
c. Sketch one period of the velocity function for $t \geq 0$.
d. Determine the times when the velocity is 0 over one period.
e. Find the acceleration function. Answer:
f. Sketch one period of the acceleration function for $t \geq 0$.

Answer: a.

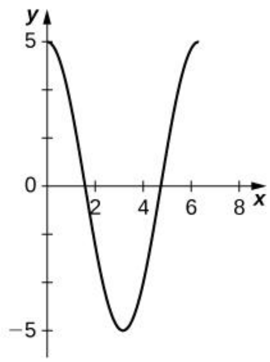


b. $v(t) = 5\sin t$

c.



d. $0, \pi, 2\pi$ e. $a(t) = 5\cos t$ f.



203. The number of hamburgers sold at a fast-food restaurant in Pasadena, California, is given by $y = 10 + 5\sin x$ where y is the number of hamburgers sold and x represents the number of hours after the restaurant opened at 11 a.m. until 11 p.m., when the store closes. Find y' and determine the intervals where the number of burgers being sold is increasing.

Answer: $y' = 5\cos(x)$, increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$, and $\left(\frac{7\pi}{2}, 12\right)$

204. **[T]** The amount of rainfall per month in Phoenix, Arizona, can be approximated by $y(t) = 0.5 + 0.3\cos t$, where t is months since January. Find y' and use a calculator to determine the intervals where the amount of rain falling is decreasing.

Answer: $y'(t) = -0.3\sin t$, decreasing on $(0, \pi)$ and $(2\pi, 3\pi)$

For the following exercises, use the quotient rule to derive the given equations.

205. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Answer: This is a proof; therefore, no answer is provided.

206. $\frac{d}{dx}(\sec x) = \sec x \tan x$

Answer: This is a proof; therefore, no answer is provided

207. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Answer: This is a proof; therefore, no answer is provided

208. Use the definition of derivative and the identity $\cos(x+h) = \cos x \cos h - \sin x \sin h$ to prove that $\frac{d(\cos x)}{dx} = -\sin x.$

Answer: This is a proof; therefore, no answer is provided`

For the following exercises, find the requested higher-order derivative for the given functions.

209. $\frac{d^3y}{dx^3}$ of $y = 3\cos x$

Answer: $3\sin x$

210. $\frac{d^2y}{dx^2}$ of $y = 3\sin x + x^2 \cos x$

Answer: $y = -(x^2 - 2)\cos x - (4x + 3)\sin x$

211. $\frac{d^4y}{dx^4}$ of $y = 5\cos x$

Answer: $5\cos x$

212. $\frac{d^2y}{dx^2}$ of $y = \sec x + \cot x$

Answer: $\sec^3 x + \tan^2(x)\sec(x) + 2\cot(x)\csc^2(x)$

213. $\frac{d^3y}{dx^3}$ of $y = x^{10} - \sec x$

Answer: $720x^7 - 5\tan(x)\sec^3(x) - \tan^3(x)\sec(x)$