

**Chapter 3**  
**Derivatives**  
**3.8 Implicit Differentiation**

**Section Exercises**

For the following exercises, use implicit differentiation to find  $\frac{dy}{dx}$ .

300.  $x^2 - y^2 = 4$

Answer:  $\frac{dy}{dx} = \frac{x}{y}$

301.  $6x^2 + 3y^2 = 12$

Answer:  $\frac{dy}{dx} = \frac{-2x}{y}$

302.  $x^2y = y - 7$

Answer:  $\frac{dy}{dx} = \frac{2xy}{1 - x^2}$

303.  $3x^3 + 9xy^2 = 5x^3$

Answer:  $\frac{dy}{dx} = \frac{x}{3y} - \frac{y}{2x}$

304.  $xy - \cos(xy) = 1$

Answer:  $\frac{dy}{dx} = \frac{-y - y \sin(xy)}{x + x \sin(xy)} = -\frac{y}{x}$

305.  $y\sqrt{x+4} = xy + 8$

Answer:  $\frac{dy}{dx} = \frac{y - \frac{y}{2\sqrt{x+4}}}{\sqrt{x+4} - x}$

306.  $-xy - 2 = \frac{x}{7}$

Answer:  $\frac{dy}{dx} = -\frac{7y+1}{7x}$

307.  $y \sin(xy) = y^2 + 2$

Answer:  $\frac{dy}{dx} = \frac{y^2 \cos(xy)}{2y - \sin(xy) - xy \cos xy}$

308.  $(xy)^2 + 3x = y^2$

Answer:  $\frac{dy}{dx} = \frac{2xy^2 + 3}{2y - 2x^2y}$

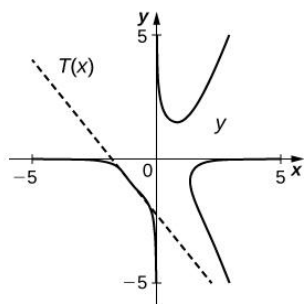
309.  $x^3y + xy^3 = -8$

Answer:  $\frac{dy}{dx} = \frac{-3x^2y - y^3}{x^3 + 3xy^2}$

**For the following exercises, find the equation of the tangent line to the graph of the given equation at the indicated point. Use a calculator or computer software to graph the function and the tangent line.**

310. [T]  $x^4y - xy^3 = -2, (-1, -1)$

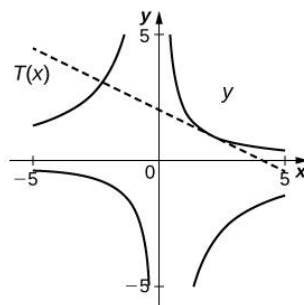
Answer:



$$y = -\frac{5}{4}x - \frac{9}{4}$$

311. [T]  $x^2y^2 + 5xy = 14, (2, 1)$

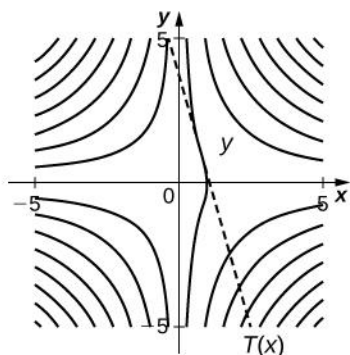
Answer:



$$y = -\frac{1}{2}x + 2$$

312. [T]  $\tan(xy) = y, \left(\frac{\pi}{4}, 1\right)$

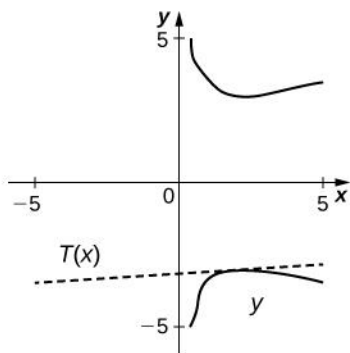
Answer:



$$y = \frac{4}{2-\pi}x + \frac{2-2\pi}{2-\pi}$$

313. [T]  $xy^2 + \sin(\pi y) - 2x^2 = 10, (2, -3)$

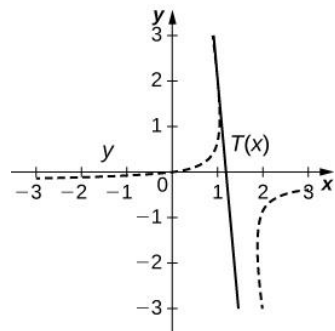
Answer:



$$y = \frac{1}{\pi+12}x - \frac{3\pi+38}{\pi+12}$$

314. [T]  $\frac{x}{y} + 5x - 7 = -\frac{3}{4}y, (1, 2)$

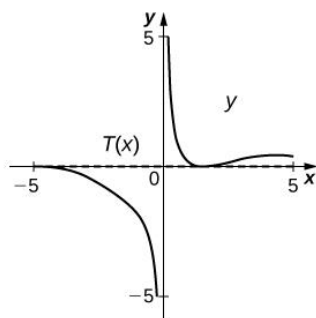
Answer:



$$y = 13 - 11x$$

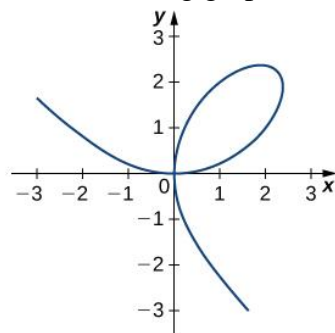
315. [T]  $xy + \sin(x) = 1, \left(\frac{\pi}{2}, 0\right)$

Answer:



$y = 0$

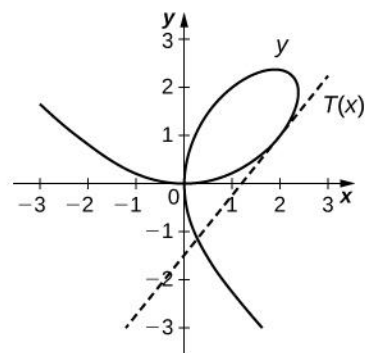
316. [T] The graph of a folium of Descartes with equation  $2x^3 + 2y^3 - 9xy = 0$  is given in the following graph.



- Find the equation of the tangent line at the point  $(2,1)$ . Graph the tangent line along with the folium.
- Find the equation of the normal line to the tangent line in a. at the point  $(2,1)$ .

Answer:

a.  $y = \frac{5}{4}x - \frac{3}{2}$



b.  $y = \frac{-4}{5}x + \frac{13}{5}$

317. For the equation  $x^2 + 2xy - 3y^2 = 0$ ,

- Find the equation of the normal to the tangent line at the point  $(1,1)$ .
- At what other point does the normal line in a. intersect the graph of the equation?

Answer: a.  $y = -x + 2$  b.  $(3, -1)$

318. Find all points on the graph of  $y^3 - 27y = x^2 - 90$  at which the tangent line is vertical.

Answer:  $(\pm 6, 3), (\pm 12, -3)$

319. For the equation  $x^2 + xy + y^2 = 7$ ,

- Find the  $x$ -intercept(s).
- Find the slope of the tangent line(s) at the  $x$ -intercept(s).
- What does the value(s) in b. indicate about the tangent line(s)?

Answer: a.  $(\pm\sqrt{7}, 0)$  b.  $-2$  c. They are parallel since the slope is the same at both intercepts.

320. Find the equation of the tangent line to the graph of the equation  $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{6}$  at

the point  $\left(0, \frac{1}{2}\right)$ .

Answer:  $y = -\frac{\sqrt{3}}{2}x + \frac{1}{2}$

321. Find the equation of the tangent line to the graph of the equation  $\tan^{-1}(x + y) = x^2 + \frac{\pi}{4}$  at

the point  $(0, 1)$ .

Answer:  $y = -x + 1$

322. Find  $y'$  and  $y''$  for  $x^2 + 6xy - 2y^2 = 3$ .

Answer:  $y' = -\frac{x + 3y}{3x - 2y}$ ,  $y'' = \frac{11(y - xy')}{(3x - 2y)^2} = \frac{11(x^2 + 6xy - 2y^2)}{(3x - 2y)^3}$

323. [T] The number of cell phones produced when  $x$  dollars is spent on labor and  $y$  dollars is spent on capital invested by a manufacturer can be modeled by the equation  $60x^{3/4}y^{1/4} = 3240$ .

- Find  $\frac{dy}{dx}$  and evaluate at the point  $(81, 16)$ .
- Interpret the result of a.

Answer: a.  $-0.5926$  b. When \$81 is spent on labor and \$16 is spent on capital, the amount spent on capital is decreasing by \$0.5926 per \$1 spent on labor.

324. [T] The number of cars produced when  $x$  dollars is spent on labor and  $y$  dollars is spent on capital invested by a manufacturer can be modeled by the equation  $30x^{1/3}y^{2/3} = 360$ . (Both  $x$  and  $y$  are measured in thousands of dollars.)

a. Find  $\frac{dy}{dx}$  and evaluate at the point  $(27, 8)$ .

b. Interpret the result of a.

Answer: a.  $-0.1481$  b. When \$27,000 is spent on labor and \$8,000 is spent on capital, the amount spent on capital is decreasing by \$148.10 (\$0.1481 thousand) per \$1,000 spent on labor.

325. The volume of a right circular cone of radius  $x$  and height  $y$  is given by  $V = \frac{1}{3}\pi x^2 y$ .

Suppose that the volume of the cone is  $85\pi \text{ cm}^3$ . Find  $\frac{dy}{dx}$  when  $x = 4$  and  $y = 16$ .

Answer:  $-8$

**For the following exercises, consider a closed rectangular box with a square base with side  $x$  and height  $y$ .**

326. Find an equation for the surface area of the rectangular box,  $S(x, y)$ .

Answer:  $S(x, y) = 2x^2 + 4xy$

327. If the surface area of the rectangular box is 78 square feet, find  $\frac{dy}{dx}$  when  $x = 3$  feet and  $y = 5$  feet.

Answer:  $-2.67$

**For the following exercises, use implicit differentiation to determine  $y'$ . Does the answer agree with the formulas we have previously determined?**

328.  $x = \sin y$

Answer:  $y' = \frac{1}{\sqrt{1-x^2}}$

329.  $x = \cos y$

Answer:  $y' = -\frac{1}{\sqrt{1-x^2}}$

330.  $x = \tan y$

Answer:  $y' = \frac{1}{1+x^2}$