Chapter 3
Derivatives
3.3 Differentiation Rules

Section Exercises

For the following exercises, find $f'(x)$ for each function.

106. $f(x) = x^7 + 10$

Answer: $f'(x) = 7x^6$

107. $f(x) = 5x^3 - x + 1$

Answer: $f'(x) = 15x^2 - 1$

108. $f(x) = 4x^2 - 7x$

Answer: $f'(x) = 8x - 7$

109. $f(x) = 8x^4 + 9x^2 - 1$

Answer: $f'(x) = 32x^3 + 18x$

110. $f(x) = x^4 + \dfrac{2}{x}$

Answer: $f'(x) = 4x^3 - \dfrac{2}{x^2}$

111. $f(x) = 3x\left(18x^4 + \dfrac{13}{x+1}\right)$

Answer: $f'(x) = 270x^4 + \dfrac{39}{(x+1)^2}$

112. $f(x) = (x+2)(2x^2 - 3)$

Answer: $f'(x) = 6x^2 + 8x - 3$

113. $f(x) = x^2\left(\dfrac{2}{x^2} + \dfrac{5}{x^3}\right)$

Answer: $f'(x) = \dfrac{-5}{x^2}$

114. $f(x)=\frac{x^3+2x^2-4}{3}$

Answer: $f'(x)=x^2+\frac{4}{3}x$

115. $f(x)=\frac{4x^3-2x+1}{x^2}$

Answer: $f'(x)=\frac{4x^4+2x^2-2x}{x^4}$

116. $f(x)=\frac{x^2+4}{x^2-4}$

Answer: $f'(x)=\frac{-16x}{\left(x^2-4\right)^2}$

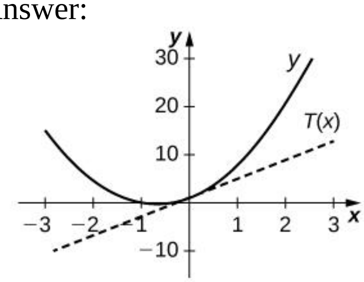
117. $f(x)=\frac{x+9}{x^2-7x+1}$

Answer: $f'(x)=\frac{-x^2-18x+64}{\left(x^2-7x+1\right)^2}$

For the following exercises, find the equation of the tangent line $T(x)$ to the graph of the given function at the indicated point. Use a graphing calculator to graph the function and the tangent line.

118. **[T]** $y=3x^2+4x+1$ at $(0,\ 1)$

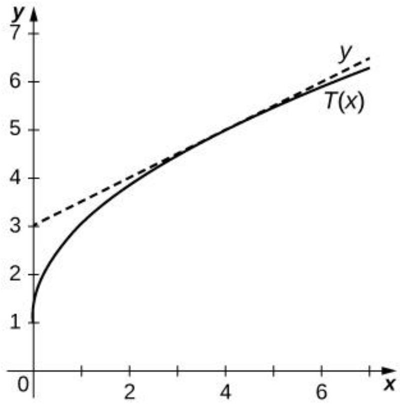
Answer:



$T(x)=4x+1$

119. **[T]** $y = 2\sqrt{x} + 1$ at $(4,\ 5)$

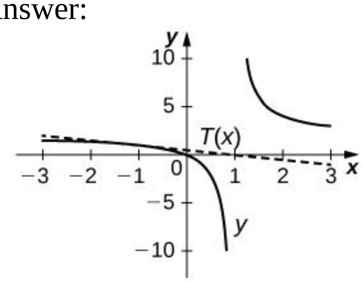
Answer:



$T(x) = \frac{1}{2}x + 3$

120. **[T]** $y = \frac{2x}{x-1}$ at $(-1,\ 1)$

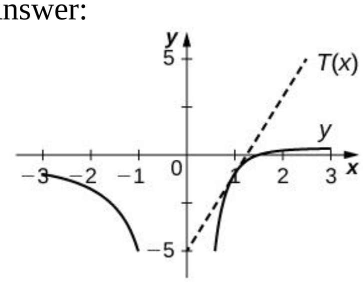
Answer:



$T(x) = -\frac{1}{2}x + \frac{1}{2}$

121. **[T]** $y = \frac{2}{x} - \frac{3}{x^2}$ at $(1,\ -1)$

Answer:



$T(x) = 4x - 5$

For the following exercises, assume that $f(x)$ and $g(x)$ are both differentiable functions for all $x.$ Find the derivative of each of the functions $h(x).$

122. $h(x)=4f(x)+\dfrac{g(x)}{7}$

Answer: $h'(x)=4f'(x)+\dfrac{g'(x)}{7}$

123. $h(x)=x^3 f(x)$

Answer: $h'(x)=3x^2 f(x)+x^3 f'(x)$

124. $h(x)=\dfrac{f(x)g(x)}{2}$

Answer: $h'(x)=\dfrac{1}{2}\left(f'(x)g(x)+g'(x)f(x)\right)$

125. $h(x)=\dfrac{3f(x)}{g(x)+2}$

Answer: $h'(x)=\dfrac{3f'(x)\left(g(x)+2\right)-3f(x)g'(x)}{\left(g(x)+2\right)^2}$

For the following exercises, assume that $f(x)$ and $g(x)$ are both differentiable functions with values as given in the following table. Use the following table to calculate the following derivatives.

$\boldsymbol{x}$	1	2	3	4
$\boldsymbol{f(x)}$	3	5	–2	0
$\boldsymbol{g(x)}$	2	3	–4	6
$\boldsymbol{f'(x)}$	–1	7	8	–3
$\boldsymbol{g'(x)}$	4	1	2	9

126. Find $h'(1)$ if $h(x)=xf(x)+4g(x).$

Answer: 18

127. Find $h'(2)$ if $h(x)=\dfrac{f(x)}{g(x)}.$

Answer: $\dfrac{16}{9}$

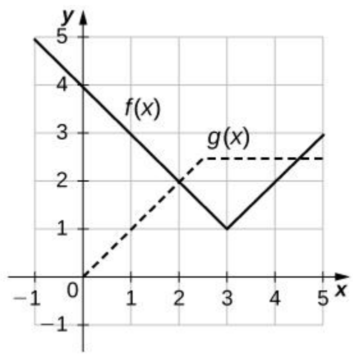
128. Find $h'(3)$ if $h(x) = 2x + f(x)g(x)$.

Answer: -34

129. Find $h'(4)$ if $h(x) = \frac{1}{x} + \frac{g(x)}{f(x)}$.

Answer: Undefined

For the following exercises, use the following figure to find the indicated derivatives, if they exist.



130. Let $h(x) = f(x) + g(x)$. Find

a. $h'(1)$,

b. $h'(3)$, and

c. $h'(4)$.

Answers: a. 0, b. does not exist, c. 1

131. Let $h(x) = f(x)g(x)$. Find

a. $h'(1)$,

b. $h'(3)$, and

c. $h'(4)$.

Answers: a. 2, b. does not exist, c. 2.5

132. Let $h(x) = \frac{f(x)}{g(x)}$. Find

a. $h'(1)$,

b. $h'(3)$, and

c. $h'(4)$.

Answers: a. -4, b. does not exist, c. $\frac{2}{5}$

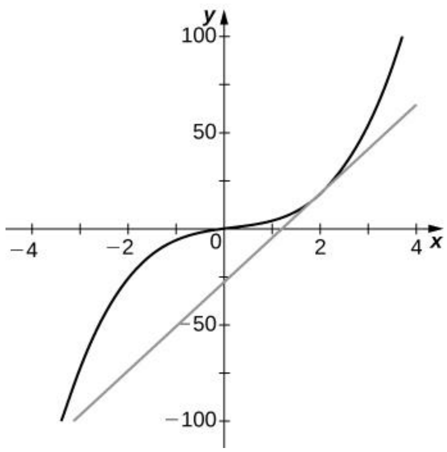
For the following exercises,

a. evaluate $f'(a)$, **and**

b. graph the function $f(x)$ **and the tangent line at** $x = a$.

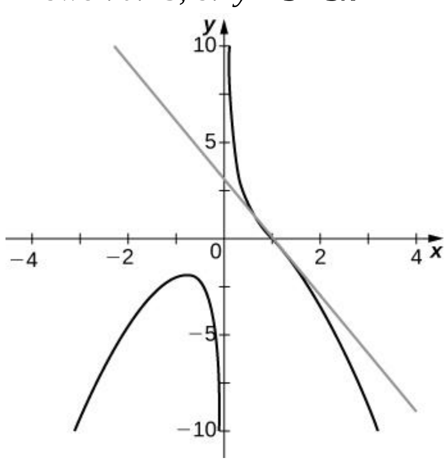
133. **[T]** $f(x) = 2x^3 + 3x - x^2,\ a = 2$

Answer: a. 23, b. $y = 23x - 28$



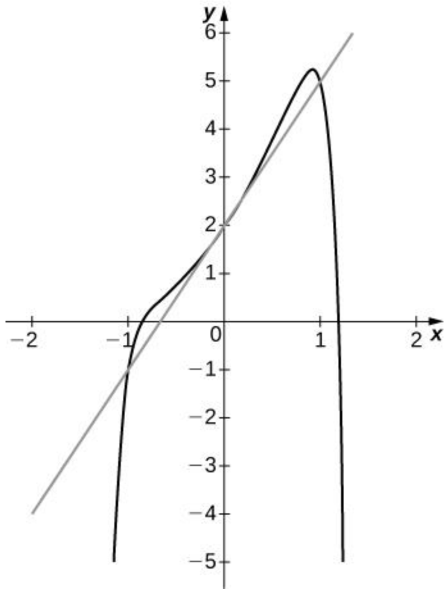
134. **[T]** $f(x) = \frac{1}{x} - x^2,\ a = 1$

Answer: a. -3, b. $y = 3 - 3x$



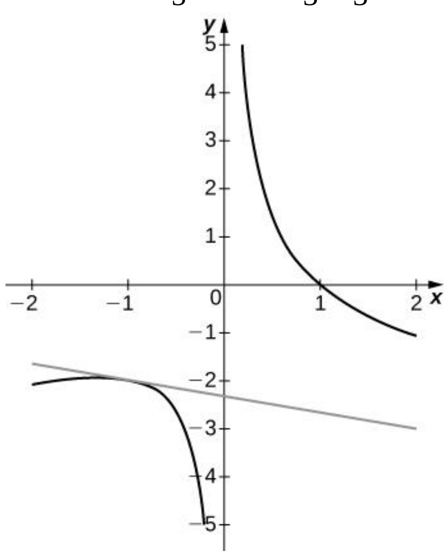
135. **[T]** $f(x) = x^2 - x^{12} + 3x + 2,\ a = 0$

Answer: a. 3, b. $y = 3x + 2$



136. **[T]** $f(x) = \frac{1}{x} - x^{2/3},\ a = -1$

Answer: a. $-\frac{1}{3}$, b. $y = \frac{x}{3} - \frac{7}{3}$



137. Find the equation of the tangent line to the graph of $f(x) = 2x^3 + 4x^2 - 5x - 3$ at $x = -1$.

Answer: $y = -7x - 3$

138. Find the equation of the tangent line to the graph of $f(x) = x^2 + \frac{4}{x} - 10$ at $x = 8$.

Answer: $y = \frac{255}{16}x - 73$

139. Find the equation of the tangent line to the graph of $f(x) = (3x - x^2)(3 - x - x^2)$ at $x = 1$.

Answer: $y = -5x + 7$

140. Find the point on the graph of $f(x) = x^3$ such that the tangent line at that point has an x intercept of 6.

Answer: $(9,\ 729)$

141. Find the equation of the line passing through the point $P\ (3,\ 3)$ and tangent to the graph of $f(x) = \frac{6}{x-1}$.

Answer: $y = -\frac{3}{2}x + \frac{15}{2}$

142. Determine all points on the graph of $f(x) = x^3 + x^2 - x - 1$ for which the slope of the tangent line is

a. horizontal
b. -1.

Answer: a. $(-1,\ 0)$, $\left(\frac{1}{3},\ \frac{-32}{27}\right)$ b. $(0,\ -1)$, $\left(\frac{-2}{3},\ \frac{-5}{27}\right)$

143. Find a quadratic polynomial such that $f(1) = 5$, $f'(1) = 3$ and $f''(1) = -6$.

Answer: $y = -3x^2 + 9x - 1$

144. A car driving along a freeway with traffic has traveled $s(t) = t^3 - 6t^2 + 9t$ meters in t seconds.

a. Determine the time in seconds when the velocity of the car is 0.
b. Determine the acceleration of the car when the velocity is 0.

Answer: a. $t = 1\text{s}$ and $t = 3\text{s}$ b. $\pm 6\ \text{m/s}^2$

145. **[T]**A herring swimming along a straight line has traveled $s(t) = \frac{t^2}{t^2 + 2}$ feet in t seconds. Determine the velocity of the herring when it has traveled 3 seconds.

Answer: $\frac{12}{121}$ or 0.0992 ft/s

146. The population in millions of arctic flounder in the Atlantic Ocean is modeled by the function $P(t)=\frac{8t+3}{0.2t^2+1}$, where t is measured in years.

a. Determine the initial flounder population.

b. Determine $P'(10)$ and briefly interpret the result.

Answer: a. 3 million b. At year 10, the fish population is decreasing at a rate of approximately 0.3719 million per year or 371,900 per year.

147. **[T]** The concentration of antibiotic in the bloodstream t hours after being injected is given by the function $C(t)=\frac{2t^2+t}{t^3+50}$, where C is measured in milligrams per liter of blood.

a. Find the rate of change of $C(t)$.

b. Determine the rate of change for $t=8,\ 12,\ 24,$ and 36

c. Briefly describe what seems to be occurring as the number of hours increases.

Answer: a. $\frac{-2t^4-2t^3+200t+50}{\left(t^3+50\right)^2}$ b. -0.02395 mg/L-hr, -0.01344 mg/L-hr, -0.003566 mg/L-hr, -0.001579 mg/L-hr c. The rate at which the concentration of drug in the bloodstream decreases is slowing to 0 as time increases.

148. A book publisher has a cost function given by $C(x)=\frac{x^3+2x+3}{x^2}$, where x is the number of copies of a book in thousands and C is the cost, per book, measured in dollars. Evaluate $C'(2)$ and explain its meaning.

Answer: If the publisher publishes more than 2000 books, the rate at which the cost of publication decreases is \$0.25

149. **[T]** According to Newton's law of universal gravitation, the force F between two bodies of constant mass m_1 and m_2 is given by the formula $F=\frac{G\,m_1m_2}{d^2}$, where G is the gravitational constant and d is the distance between the bodies.

a. Suppose that $G,\ m_1,$ and m_2 are constants. Find the rate of change of force F with respect to distance d.

b. Find the rate of change of force F with gravitational constant $G=6.67\times10^{-11}$ $\mathrm{Nm}^2/\mathrm{kg}^2$, on two bodies 10 meters apart, each with a mass of 1000 kilograms.

Answer: a. $F'(d)=\frac{-2G\,m_1m_2}{d^3}$ b. -1.33×10^{-7} N/m

Student Project
Formula One Grandstands

1. Physicists have determined that drivers are most likely to lose control of their cars as they are coming into a turn, at the point where the slope of the tangent line is 1. Find the $(x,\ y)$ coordinates of this point near the turn.

Answer: Set $f'(x) = 1$ and solve. $(x, y) = (-2, 2)$

2. Find the equation of the tangent line to the curve at this point.

Answer: $y = x + 4$

3. To determine whether the spectators are in danger in this scenario, find the *x*-coordinate of the point where the tangent line crosses the line $y = 2.8$. Is this point safely to the right of the grandstand? Or are the spectators in danger?

Answer: $x = -1.2$ This is safely to the right of the grandstand.

4. What if a driver loses control earlier than the physicists project? Suppose a driver loses control at the point $(-2.5,\ 0.625)$. What is the slope of the tangent line at this point?

Answer: $f'\left(-2\frac{1}{2}\right) = 4\frac{3}{4}$

5. If a driver loses control as described in part 4, are the spectators safe?

Answer: Under this scenario, the spectators are not safe. When $y = 2.8$, the *x*-coordinate of the tangent line is approximately -2.04. A car traveling on that path would hit the grandstand.

6. Should you proceed with the current design for the grandstand, or should the grandstands be moved?

Answer: Move the grandstands.