Chapter 2
Limits
2.1 A Preview of Calculus

Section Exercises

For the following exercises, points $P(1,2)$ and $Q(x,y)$ are on the graph of the function $f(x)=x^2+1$.

1. **[T]** Complete the following table with the appropriate values: *y*-coordinate of *Q*, the point $Q(x,y)$, and the slope of the secant line passing through points *P* and *Q*. Round your answer to eight significant digits.

x	y	$Q(x,y)$	m_{sec}
1.1	a.	e.	i.
1.01	b.	f.	j.
1.001	c.	g.	k.
1.0001	d.	h.	l.

Answer: a. 2.2100000; b. 2.0201000; c. 2.0020010; d. 2.0002000; e. (1.1000000, 2.2100000); f. (1.0100000, 2.0201000); g. (1.0010000, 2.0020010); h. (1.0001000, 2.0002000); i. 2.1000000; j. 2.0100000; k. 2.0010000; l. 2.0001000

2. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the line tangent to *f* at $x=1$.

Answer: 2

3. Use the value in the preceding exercise to find the equation of the tangent line at point *P*. Graph $f(x)$ and the tangent line.

Answer: $y=2x$

For the following exercises, points $P(1,1)$ and $Q(x,y)$ are on the graph of the function $f(x)=x^3$.

4. **[T]** Complete the following table with the appropriate values: *y*-coordinate of *Q*, the point $Q(x,y)$, and the slope of the secant line passing through points *P* and *Q*. Round your answer to eight significant digits.

x	y	$Q(x,y)$	m_{sec}
1.1	a.	e.	i.
1.01	b.	f.	j.
1.001	c.	g.	k.
1.0001	d.	h.	l.

Answer: a. 1.3310000; b. 1.0303010; c. 1.0030030; d. 1.0003000; e. (1.1000000, 1.331000); f. (1.0100000, 1.0303010); g. (1.0010000, 1.003030); h. (1.0001000, 1.0003000); i. 3.3100000; j. 3.0301000; k. 3.0030010; l. 3.0003000

5. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to *f* at $x=1$.

Answer: 3

6. Use the value in the preceding exercise to find the equation of the tangent line at point *P*. Graph $f(x)$ and the tangent line.

Answer: $y = 3x - 2$

For the following exercises, points $P(4,2)$ and $Q(x,y)$ are on the graph of the function $f(x) = \sqrt{x}$.

7. **[T]** Complete the following table with the appropriate values: *y*-coordinate of *Q*, the point $Q(x,y)$, and the slope of the secant line passing through points *P* and *Q*. Round your answer to eight significant digits.

x	y	$Q(x,y)$	m_{sec}
4.1	a.	e.	i.
4.01	b.	f.	j.
4.001	c.	g.	k.
4.0001	d.	h.	l.

Answer: a. 2.0248457; b. 2.0024984; c. 2.0002500; d. 2.0000250; e. (4.1000000,2.0248457); f. (4.0100000,2.0024984); g. (4.0010000,2.0002500); h. (4.00010000,2.0000250); i. 0.24845673; j. 0.24984395; k. 0.24998438; l. 0.24999844

8. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to *f* at $x = 4$.

Answer: 0.25

9. Use the value in the preceding exercise to find the equation of the tangent line at point *P*.

Answer: $y = \frac{x}{4} + 1$

For the following exercises, points $P(1.5,0)$ and $Q(\phi,y)$ are on the graph of the function $f(\phi) = \cos(\pi\phi)$.

10. **[T]** Complete the following table with the appropriate values: *y*-coordinate of *Q*, the point $Q(\phi,y)$, and the slope of the secant line passing through points *P* and *Q*. Round your answer to eight significant digits.

X	y	$Q(\phi,y)$	m_{sec}
1.4	a.	e.	i.
1.49	b.	f.	j.
1.499	c.	g.	k.
1.4999	d.	h.	l.

Answer: a. –0.30901699; b. –0.031410759; c. -0.0031415875; d. -0.00031415926; e. (1.4000000,–0.30901700); f. (1.4900000,–0.031410759); g. (1.4990000,–0.0031415875); h. (1.4999000,–0.00031415926); i. 3.0901699; j. 3.1410759; k. 3.1415875; l. 3.1415926

11. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to *f* at $\phi = 1.5$.

Answer: π

12. Use the value in the preceding exercise to find the equation of the tangent line at point *P*.

Answer: $y = \pi x - 1.5\pi$

For the following exercises, points $P(-1,-1)$ and $Q(x,y)$ are on the graph of the function $f(x) = \frac{1}{x}$.

13. **[T]** Complete the following table with the appropriate values: *y*-coordinate of *Q*, the point $Q(x,y)$, and the slope of the secant line passing through points *P* and *Q*. Round your answer to eight significant digits.

x	y	$Q(x,y)$	m_{sec}
−1.05	a.	e.	i.
−1.01	b.	f.	j.
−1.005	c.	g.	k.
−1.001	d.	h.	l.

Answer: a. −0.95238095; b. −0.99009901; c. −0.99502488; d. −0.99900100; e. (–1.0500000,–0.95238095); f. (–1.0100000,–0.9909901); g. (–1.0050000,–0.99502488); h. (1.0010000,–0.99900100); i. −0.95238095; j. −0.99009901; k. −0.99502488; l. −0.99900100

14. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the line tangent to *f* at $x = -1$.

Answer: −1

15. Use the value in the preceding exercise to find the equation of the tangent line at point *P*.

Answer: $y = -x - 2$

For the following exercises, the position function of a ball dropped from the top of a 200-meter tall building is given by $s(t) = 200 - 4.9t^2$, where position *s* is measured in meters and time *t* is measured in seconds. Round your answer to eight significant digits.

16. **[T]** Compute the average velocity of the ball over the given time intervals.
    a. $[4.99, 5]$
    b. $[5, 5.01]$
    c. $[4.999, 5]$
    d. $[5, 5.001]$

Answer: a. –48.951000 m/sec; b. –49.049000 m/sec; c. –48.995100 m/sec; d. –49.000490 m/sc

17. Use the preceding exercise to guess the instantaneous velocity of the ball at $t = 5$ sec.

Answer: –49 m/sec (velocity of the ball is 49 m/sec downward)

For the following exercises, consider a stone tossed into the air from ground level with an initial velocity of 15 m/sec. Its height in meters at time *t* seconds is $h(t) = 15t - 4.9t^2$.

18. **[T]** Compute the average velocity of the stone over the given time intervals.
    a. $[1,1.05]$
    b. $[1,1.01]$
    c. $[1,1.005]$
    d. $[1,1.001]$

Answer: a. 4.9950000 m/sec; b. 5.1510000 m/sec; c. 5.1755000 m/sec; d. 5.1951000 m/sec

19. Use the preceding exercise to guess the instantaneous velocity of the stone at $t = 1$ sec.

Answer: 5.2 m/sec

For the following exercises, consider a rocket shot into the air that then returns to Earth. The height of the rocket in meters is given by $h(t) = 600 + 78.4t - 4.9t^2$, where *t* is measured in seconds.

20. **[T]** Compute the average velocity of the rocket over the given time intervals.
    a. $[9,9.01]$
    b. $[8.99,9]$
    c. $[9,9.001]$
    d. $[8.999,9]$

Answer: a. −9.8490000 m/sec; b. −9.7510000 m/sec; c. −9.8049000 m/sec; d. −9.7951000 m/sec

21. Use the preceding exercise to guess the instantaneous velocity of the rocket at $t = 9$ sec.

Answer: −9.8 m/sec

For the following exercises, consider an athlete running a 40-m **dash. The position of the athlete is given by $d(t) = \frac{t^3}{6} + 4t$, where *d* is the position in meters and *t* is the time elapsed, measured in seconds.**

22. **[T]** Compute the average velocity of the runner over the given time intervals.
    a. $[1.95, 2.05]$
    b. $[1.995, 2.005]$
    c. $[1.9995, 2.0005]$
    d. $[2, 2.00001]$

Answer: a. 6.0004167 m/sec; b) 6.0000042 m/sec; c. 6.0000000 m/sec; d. 6.0000000 m/sec

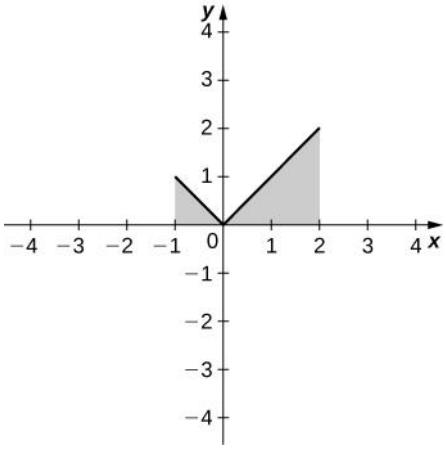
23. Use the preceding exercise to guess the instantaneous velocity of the runner at $t = 2$ sec.

Answer: 6 m/sec

For the following exercises, consider the function $f(x) = |x|$.

24. Sketch the graph of *f* over the interval $[-1, 2]$ and shade the region above the *x*-axis.

Answer:



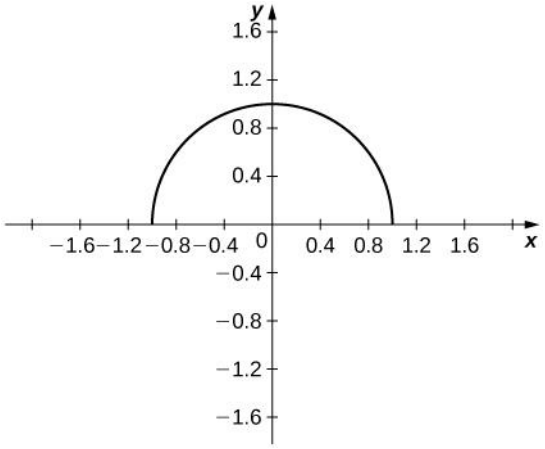
25. Use the preceding exercise to find the approximate value of the area between the *x*-axis and the graph of *f* over the interval $[-1, 2]$ using rectangles. For the rectangles, use the square units, and approximate both above and below the lines. Use geometry to find the exact answer.

Answer: Under, 1 unit^2; over: 4 unit^2. The exact area of the two triangles is $\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = 2.5 \text{ units}^2$.

For the following exercises, consider the function $f(x) = \sqrt{1 - x^2}$. (*Hint*: This is the upper half of a circle of radius 1 positioned at $(0, 0)$.)

26. Sketch the graph of *f* over the interval $[-1, 1]$.

Answer:



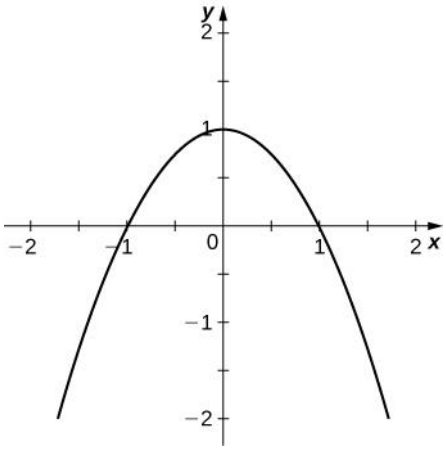
27. Use the preceding exercise to find the approximate area between the *x*-axis and the graph of *f* over the interval $[-1,1]$ using rectangles. For the rectangles, use squares 0.4 by 0.4 units, and approximate both above and below the lines. Use geometry to find the exact answer.

Answer: Under, 0.96 unit^2; over, 1.92 unit^2. The exact area of the semicircle with radius 1 is $\frac{\pi(1)^2}{2} = \frac{\pi}{2}$ unit^2.

For the following exercises, consider the function $f(x) = -x^2 + 1$.

28. Sketch the graph of *f* over the interval $[-1,1]$.

Answer:



29. Approximate the area of the region between the *x*-axis and the graph of *f* over the interval $[-1,1]$.

Answer: Approximately 1.3333333 unit^2