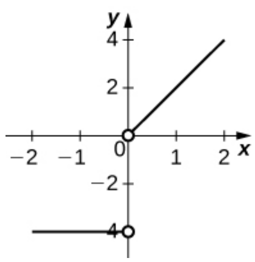
Review Exercises

367. False.
369. False
371. $\frac{1}{2\sqrt{x+4}}$
373. $9x^2 + \frac{8}{x^3}$
375. $e^{\sin x}\cos x$
377. $x\sec^2(x) + 2x\cos(x) + \tan(x) - x^2\sin(x)$
379. $\frac{1}{4}\left(\frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)\right)$
381. $\cos x \cdot (\ln x + 1) - x\ln(x)\sin x$
383. $4^x(\ln 4)^2 + 2\sin x + 4x\cos x - x^2\sin x$
385. $T = (2+e)x - 2$
387.



389. $w'(3) = -\frac{2.9\pi}{6}$. At 3 a.m. the tide is decreasing at a rate of 1.514 ft/hr.
391. -7.5. The wind speed is decreasing at a rate of 7.5 mph/hr

Chapter 4

Checkpoint

4.1. $\frac{1}{72\pi}$ cm/sec, or approximately 0.0044 cm/sec
4.2. 500 ft/sec
4.3. $\frac{1}{10}$ rad/sec
4.4. -0.61 ft/sec
4.5. $L(x) = 2 + \frac{1}{12}(x-8)$; 2.00833
4.6. $L(x) = -x + \frac{\pi}{2}$
4.7. $L(x) = 1 + 4x$
4.8. $dy = 2xe^{x^2}dx$
4.9. $dy = 1.6, \quad \Delta y = 1.64$
4.10. The volume measurement is accurate to within 21.6 cm^3.
4.11. 7.6%
4.12. $x = -\frac{2}{3}, \quad x = 1$
4.13. The absolute maximum is 3 and it occurs at $x = 4$. The absolute minimum is -1 and it occurs at $x = 2$.
4.14. $c = 2$
4.15. $\frac{5}{2\sqrt{2}}$ sec

4.16. f has a local minimum at -2 and a local maximum at $3.$

4.17. f has no local extrema because f' does not change sign at $x = 1.$

4.18. f is concave up over the interval $\left(-\infty, \frac{1}{2}\right)$ and concave down over the interval $\left(\frac{1}{2}, \infty\right)$

4.19. f has a local maximum at -2 and a local minimum at $3.$

4.20. Both limits are $3.$ The line $y = 3$ is a horizontal asymptote.

4.21. Let $\varepsilon > 0.$ Let $N = \frac{1}{\sqrt{\varepsilon}}.$ Therefore, for all $x > N,$ we have $\left|3 - \frac{1}{x^2} - 3\right| = \frac{1}{x^2} < \frac{1}{N^2} = \varepsilon$ Therefore, $\lim_{x \to \infty}\left(3 - 1/x^2\right) = 3.$

4.22. Let $M > 0.$ Let $N = \sqrt{\frac{M}{3}}.$ Then, for all $x > N,$ we have $3x^2 > 3N^2 = 3\left(\sqrt{\frac{M}{3}}\right)^2 2 = \frac{3M}{3} = M$

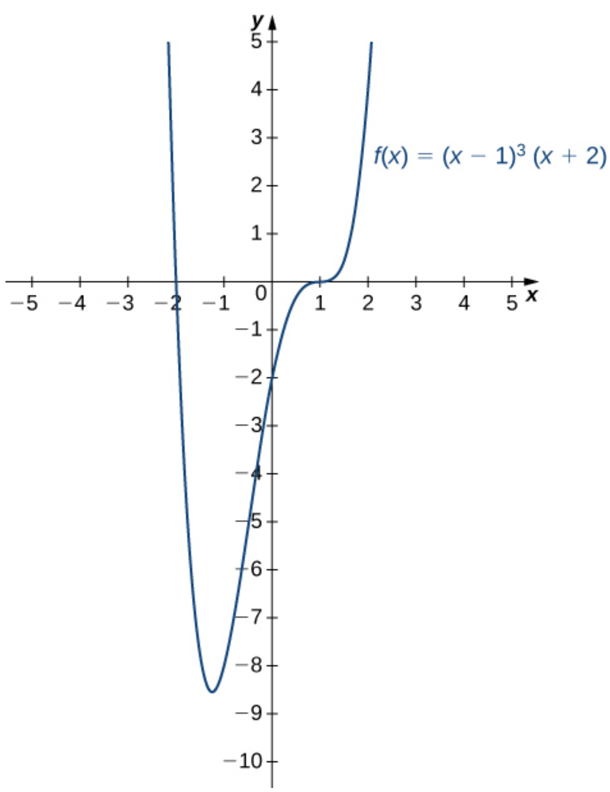
4.23. $-\infty$

4.24. $\frac{3}{5}$

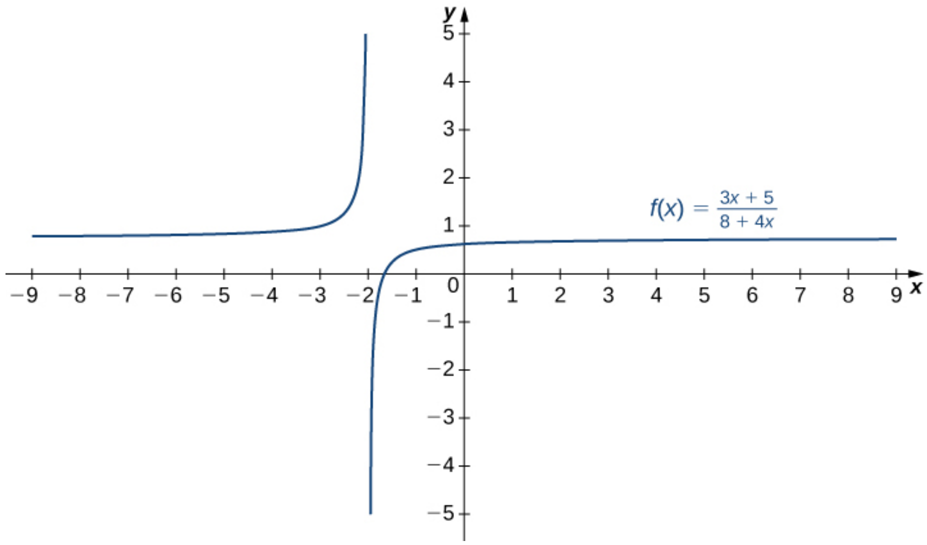
4.25. $\pm\sqrt{3}$

4.26. $\lim_{x \to \infty} f(x) = \frac{3}{5}, \quad \lim_{x \to -\infty} f(x) = -2$

4.27.



4.28.



4.29. $y = \frac{3}{2}x$

4.30. The function f has a cusp at $(0, 5)$ $\lim_{x \to 0^-} f'(x) = \infty$, $\lim_{x \to 0^+} f'(x) = -\infty$. For end behavior, $\lim_{x \to \pm\infty} f(x) = -\infty$.

4.31. The maximum area is 5000 ft^2.

4.32. $V(x) = x(20 - 2x)(30 - 2x)$. The domain is $[0, 10]$.

4.33. $T(x) = \frac{x}{6} + \frac{\sqrt{(15 - x)^2 + 1}}{2.5}$

4.34. The company should charge $\$75$ per car per day.

4.35. $A(x) = 4x\sqrt{1 - x^2}$. The domain of consideration is $[0, 1]$.

4.36. $c(x) = \frac{259.2}{x} + 0.2x^2$ dollars

4.37. 1

4.38. 0

4.39. $\lim_{x \to 0^+} \cos x = 1$. Therefore, we cannot apply L'Hôpital's rule. The limit of the quotient is ∞

4.40. 1

4.41. 0

4.42. e

4.43. 1

4.44. The function 2^x grows faster than x^{100}.

4.45. $x_1 \approx 0.33333333$, $x_2 \approx 0.347222222$

4.46. $x_1 = 2$, $x_2 = 1.75$

4.47. $x_1 \approx -1.842105263$, $x_2 \approx -1.772826920$

4.48. $x_1 = 6$, $x_2 = 8$, $x_3 = \frac{26}{3}$, $x_4 = \frac{80}{9}$, $x_5 = \frac{242}{27}$; $x^* = 9$

4.49. $-\cos x + C$

4.50. $\frac{d}{dx}(x \sin x + \cos x + C) = \sin x + x \cos x - \sin x = x \cos x$

4.51. $x^4 - \frac{5}{3}x^3 + \frac{1}{2}x^2 - 7x + C$

4.52. $y = -\frac{3}{x} + 5$

4.53. 2.93 sec, 64.5 ft

Section Exercises

1. 8

3. $\frac{13}{\sqrt{10}}$

5. $2\sqrt{3}$ ft/sec

7. The distance is decreasing at 390 mi/h.

9. The distance between them shrinks at a rate of $\frac{1320}{13} \approx 101.5$ mph.

11. $\frac{9}{2}$ ft/sec

13. It grows at a rate $\frac{4}{9}$ ft/sec

15. The distance is increasing at $\frac{(135\sqrt{26})}{26}$ ft/sec

17. $-\frac{5}{6}$ m/sec

19. 240π m^2/sec

21. $\frac{1}{2\sqrt{\pi}}$ cm

23. The area is increasing at a rate $\frac{(3\sqrt{3})}{8}$ft /sec.

25. The depth of the water decreases at $\frac{128}{125\pi}$ ft/min.

27. The volume is decreasing at a rate of $\frac{(25\pi)}{16}$ft^3/min.

29. The water flows out at rate $\frac{(2\pi)}{5}$m /min.

31. $\frac{3}{2}$ m/sec

33. $\frac{25}{19\pi}$ ft/min

35. $\frac{2}{45\pi}$ ft/min

37. The angle decreases at $\frac{400}{1681}$rad/sec.

39. 100πmi/min

41. The angle is changing at a rate of $\frac{11}{25}$rad/sec.

43. The distance is increasing at a rate of 62.50 ft/sec.

45. The distance is decreasing at a rate of 11.99 ft/sec.

47. $f'(a) = 0$

49. The linear approximation exact when $y = f(x)$ is linear or constant.

51. $L(x) = \frac{1}{2} - \frac{1}{4}(x - 2)$

53. $L(x) = 1$

55. $L(x) = 0$

57. 0.02

59. 1.9996875

61. 0.001593

63. 1; error, ~ 0.00005

65. 0.97; error, ~ 0.0006

67. $3 - \frac{1}{600}$; error, $\sim 4.632 \times 10^{-7}$
69. $dy = (\cos x - x \sin x)dx$
71. $dy = \left(\frac{x^2 - 2x - 2}{(x-1)^2}\right)dx$
73. $dy = -\frac{1}{(x+1)^2}dx, \quad -\frac{1}{16}$
75. $dy = \frac{9x^2 + 12x - 2}{2(x+1)^{3/2}}dx, \quad -0.1$
77. $dy = \left(3x^2 + 2 - \frac{1}{x^2}\right)dx, \quad 0.2$
79. $12x\,dx$
81. $4\pi r^2\,dr$
83. $-1.2\pi\ \text{cm}^3$
85. $-100\ \text{ft}^3$
91. Answers may vary
93. Answers will vary
95. No; answers will vary
97. Since the absolute maximum is the function (output) value rather than the *x* value, the answer is no; answers will vary
99. When $a = 0$
101. Absolute minimum at 3; Absolute maximum at −2.2; local minima at −2, 1; local maxima at −1, 2
103. Absolute minima at −2, 2; absolute maxima at −2.5, 2.5; local minimum at 0; local maxima at −1, 1
105. Answers may vary.
107. Answers may vary.
109. $x = 1$
111. None
113. $x = 0$
115. None
117. $x = -1, 1$
119. Absolute maximum: $x = 4, \quad y = \frac{33}{2}$; absolute minimum: $x = 1, \quad y = 3$
121. Absolute minimum: $x = \frac{1}{2}, \quad y = 4$
123. Absolute maximum: $x = 2\pi, \quad y = 2\pi$; absolute minimum: $x = 0, \quad y = 0$
125. Absolute maximum: $x = -3$; absolute minimum: $-1 \le x \le 1, \quad y = 2$
127. Absolute maximum: $x = \frac{\pi}{4}, \quad y = \sqrt{2}$; absolute minimum: $x = \frac{5\pi}{4}, \quad y = -\sqrt{2}$
129. Absolute minimum: $x = -2, \quad y = 1$
131. Absolute minimum: $x = -3, \quad y = -135$; local maximum: $x = 0, \quad y = 0$; local minimum: $x = 1, \quad y = -7$
133. Local maximum: $x = 1 - 2\sqrt{2}, \quad y = 3 - 4\sqrt{2}$; local minimum: $x = 1 + 2\sqrt{2}, \quad y = 3 + 4\sqrt{2}$
135. Absolute maximum: $x = \frac{\sqrt{2}}{2}, \quad y = \frac{3}{2}$; absolute minimum: $x = -\frac{\sqrt{2}}{2}, \quad y = -\frac{3}{2}$
137. Local maximum: $x = -2, \quad y = 59$; local minimum: $x = 1, \quad y = -130$
139. Absolute maximum: $x = 0, \quad y = 1$; absolute minimum: $x = -2, 2, \quad y = 0$
141. $h = \frac{9245}{49}\ \text{m}, \ t = \frac{300}{49}\ \text{s}$
143. The global minimum was in 1848, when no gold was produced.
145. Absolute minima: $x = 0, \quad x = 2, \quad y = 1$; local maximum at $x = 1, \quad y = 2$
147. No maxima/minima if a is odd, minimum at $x = 1$ if a is even
149. One example is $f(x) = |x| + 3, \ -2 \le x \le 2$
151. Yes, but the Mean Value Theorem still does not apply

153. $(-\infty, 0), (0, \infty)$
155. $(-\infty, -2), (2, \infty)$
157. 2 points
159. 5 points
161. $c = \frac{2\sqrt{3}}{3}$
163. $c = \frac{1}{2}, 1, \frac{3}{2}$
165. $c = 1$
167. Not differentiable
169. Not differentiable
171. Yes
173. The Mean Value Theorem does not apply since the function is discontinuous at $x = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}$.
175. Yes
177. The Mean Value Theorem does not apply; discontinuous at $x = 0$.
179. Yes
181. The Mean Value Theorem does not apply; not differentiable at $x = 0$.
183. $b = \pm 2\sqrt{c}$
185. $c = \pm \frac{1}{\pi} \cos^{-1}\left(\frac{\sqrt{\pi}}{2}\right), \quad c = \pm 0.1533$
187. The Mean Value Theorem does not apply.
189. $\frac{1}{2\sqrt{c+1}} - \frac{2}{c^3} = \frac{521}{2880};\ c = 3.133, 5.867$
191. Yes
193. It is constant.
195. It is not a local maximum/minimum because f' does not change sign
197. No
199. False; for example, $y = \sqrt{x}$.
201. Increasing for $-2 < x < -1$ and $x > 2$; decreasing for $x < -2$ and $-1 < x < 2$
203. Decreasing for $x < 1$, increasing for $x > 1$
205. Decreasing for $-2 < x < -1$ and $1 < x < 2$; increasing for $-1 < x < 1$ and $x < -2$ and $x > 2$
207. a. Increasing over $-2 < x < -1, 0 < x < 1, x > 2$, decreasing over $x < -2$, $-1 < x < 0, 1 < x < 2$; b. maxima at $x = -1$ and $x = 1$, minima at $x = -2$ and $x = 0$ and $x = 2$
209. a. Increasing over $x > 0$, decreasing over $x < 0$; b. Minimum at $x = 0$
211. Concave up on all x, no inflection points
213. Concave up on all x, no inflection points
215. Concave up for $x < 0$ and $x > 1$, concave down for $0 < x < 1$, inflection points at $x = 0$ and $x = 1$
217. Answers will vary
219. Answers will vary
221. a. Increasing over $-\frac{\pi}{2} < x < \frac{\pi}{2}$, decreasing over $x < -\frac{\pi}{2}, x > \frac{\pi}{2}$ b. Local maximum at $x = \frac{\pi}{2}$; local minimum at $x = -\frac{\pi}{2}$
223. a. Concave up for $x > \frac{4}{3}$, concave down for $x < \frac{4}{3}$ b. Inflection point at $x = \frac{4}{3}$
225. a. Increasing over $x < 0$ and $x > 4$, decreasing over $0 < x < 4$ b. Maximum at $x = 0$, minimum at $x = 4$ c. Concave up for $x > 2$, concave down for $x < 2$ d. Infection point at $x = 2$
227. a. Increasing over $x < 0$ and $x > \frac{60}{11}$, decreasing over $0 < x < \frac{60}{11}$ b. Minimum at $x = \frac{60}{11}$ c. Concave down for $x < \frac{54}{11}$, concave up for $x > \frac{54}{11}$ d. Inflection point at $x = \frac{54}{11}$
229. a. Increasing over $x > -\frac{1}{2}$, decreasing over $x < -\frac{1}{2}$ b. Minimum at $x = -\frac{1}{2}$ c. Concave up for all x d. No inflection points

231. a. Increases over $-\frac{1}{4} < x < \frac{3}{4},$ decreases over $x > \frac{3}{4}$ and $x < -\frac{1}{4}$ b. Minimum at $x = -\frac{1}{4},$ maximum at $x = \frac{3}{4}$ c. Concave up for $-\frac{3}{4} < x < \frac{1}{4},$ concave down for $x < -\frac{3}{4}$ and $x > \frac{1}{4}$ d. Inflection points at $x = -\frac{3}{4}, x = \frac{1}{4}$

233. a. Increasing for all x b. No local minimum or maximum c. Concave up for $x > 0,$ concave down for $x < 0$ d. Inflection point at $x = 0$

235. a. Increasing for all x where defined b. No local minima or maxima c. Concave up for $x < 1;$ concave down for $x > 1$ d. No inflection points in domain

237. a. Increasing over $-\frac{\pi}{4} < x < \frac{3\pi}{4},$ decreasing over $x > \frac{3\pi}{4}, x < -\frac{\pi}{4}$ b. Minimum at $x = -\frac{\pi}{4},$ maximum at $x = \frac{3\pi}{4}$ c. Concave up for $-\frac{\pi}{2} < x < \frac{\pi}{2},$ concave down for $x < -\frac{\pi}{2}, x > \frac{\pi}{2}$ d. Infection points at $x = \pm\frac{\pi}{2}$

239. a. Increasing over $x > 4,$ decreasing over $0 < x < 4$ b. Minimum at $x = 4$ c. Concave up for $0 < x < 8\sqrt[3]{2},$ concave down for $x > 8\sqrt[3]{2}$ d. Inflection point at $x = 8\sqrt[3]{2}$

241. $f > 0, f' > 0, f'' < 0$

243. $f > 0, f' < 0, f'' < 0$

245. $f > 0, f' > 0, f'' > 0$

247. True, by the Mean Value Theorem

249. True, examine derivative

251. $x = 1$

253. $x = -1, x = 2$

255. $x = 0$

257. Yes, there is a vertical asymptote

259. Yes, there is vertical asymptote

261. 0

263. ∞

265. $-\frac{1}{7}$

267. -2

269. -4

271. Horizontal: none, vertical: $x = 0$

273. Horizontal: none, vertical: $x = \pm 2$

275. Horizontal: none, vertical: none

277. Horizontal: $y = 0,$ vertical: $x = \pm 1$

279. Horizontal: $y = 0,$ vertical: $x = 0$ and $x = -1$

281. Horizontal: $y = 1,$ vertical: $x = 1$

283. Horizontal: none, vertical: none

285. Answers will vary, for example: $y = \frac{2x}{x - 1}$

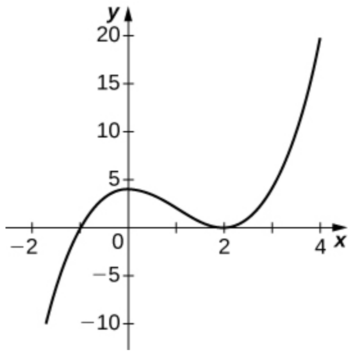
287. Answers will vary, for example: $y = \frac{4x}{x + 1}$

289. $y = 0$

291. ∞

293. $y = 3$

295.



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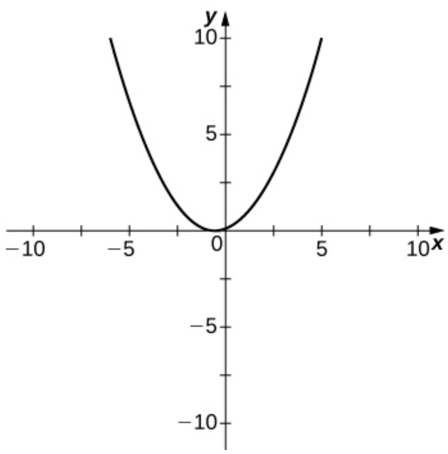

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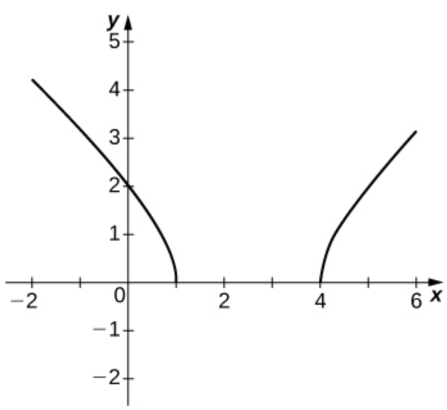

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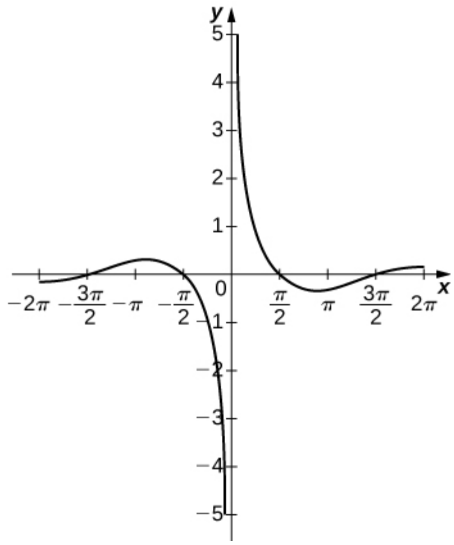
y
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4
3
2
1
0
−1
−2
−3
−4
−5
x
−2π
−3π/2
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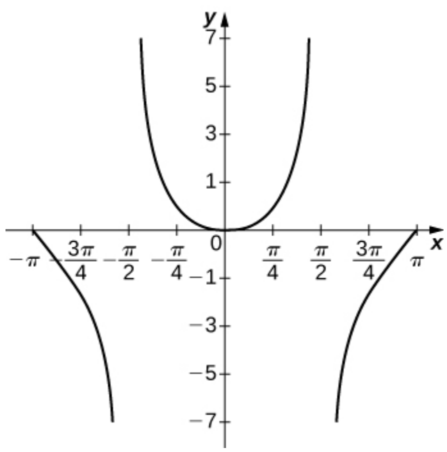
π/2
π
3π/2
2π

303.


y
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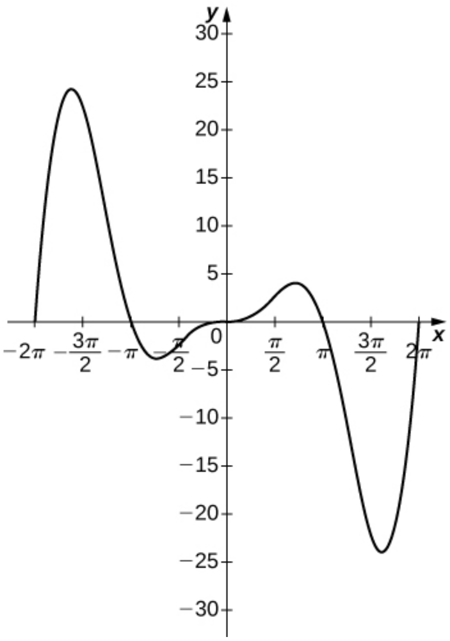
5
3
1
0
−1
−3
−5
−7
x
−π
−3π/4
−π/2
−π/4
π/4
π/2
3π/4
π

305.



307. The degree of $Q(x)$ must be greater than the degree of $P(x)$.

309. $\lim_{x \to 1^-} f(x) = -\infty$ and $\lim_{x \to 1^-} g(x) = \infty$

311. The critical points can be the minima, maxima, or neither.

313. False; $y = -x^2$ has a minimum only

315. $h = \frac{62}{3}$ in.

317. 1

319. 100 ft by 100 ft

321. 40 ft by 40 ft

323. 19.73 ft.

325. 84 bpm

327. $T(\theta) = \frac{40\theta}{3v} + \frac{40\cos\theta}{v}$

329. $v = \sqrt{\frac{b}{a}}$

331. approximately 34.02 mph

333. 4

335. 0

337. Maximal: $x = 5,\ y = 5;$ minimal: $x = 0,\ y = 10$ and $y = 0,\ x = 10$

339. Maximal: $x = 1,\ y = 9;$ minimal: none

341. $\frac{4\pi}{3\sqrt{3}}$

343. 6

345. $r = 2,\ h = 4$

347. $(2,\ 1)$

349. $(0.8351,\ 0.6974)$

351. $A = 20r - 2r^2 - \frac{1}{2}\pi r^2$

353. $C(x) = 5x^2 + \frac{32}{x}$ Differentiating, setting the derivative equal to zero and solving, we obtain $x = \sqrt[3]{\frac{2}{5}}$ and $h = \sqrt[3]{\frac{25}{4}}$.

355. $P(x) = (50 - x)(800 + 25x - 50)$

357. ∞

359. $\frac{1}{2a}$

361. $\frac{1}{na^{n-1}}$

363. Cannot apply directly; use logarithms

365. Cannot apply directly; rewrite as $\lim_{x \to 0} x^3$

367. 6

369. -2

371. -1

373. n

375. $-\frac{1}{2}$

377. $\frac{1}{2}$

379. 1

381. $\frac{1}{6}$

383. 1

385. 0

387. 0

389. -1

391. ∞

393. 0

395. $\frac{1}{e}$

397. 0

399. 1

401. 0

403. $\tan(1)$

405. 2

407. $F(x_n) = x_n - \frac{x_n{}^3 + 2x_n + 1}{3x_n{}^2 + 2}$

409. $F(x_n) = x_n - \frac{e^{x_n}}{e^{x_n}}$

411. $|c| > 0.5$ fails, $|c| \leq 0.5$ works

413. $c = \frac{1}{f'(x_n)}$

415. a. $x_1 = \frac{12}{25}, x_2 = \frac{312}{625}$; b. $x_1 = -4, x_2 = -40$

417. a. $x_1 = 1.291, x_2 = 0.8801$; b. $x_1 = 0.7071, x_2 = 1.189$

419. a. $x_1 = -\frac{26}{25}, x_2 = -\frac{1224}{625}$; b. $x_1 = 4, x_2 = 18$

421. a. $x_1 = \frac{6}{10}, x_2 = \frac{6}{10}$; b. $x_1 = 2, x_2 = 2$

423. 3.1623 or -3.1623

425. $0, -1$ or 1

427. 0

429. 0.5188 or -1.2906

431. 0

433. 4.493

435. $0.159,\ 3.146$

437. We need f to be twice continuously differentiable.

439. $x = 0$

441. $x = -1$

443. $x = 5.619$

445. $x = -1.326$

447. There is no solution to the equation.

449. It enters a cycle.

451. 0

453. -0.3513

455. Newton: 11 iterations, secant: 16 iterations

457. Newton: three iterations, secant: six iterations

459. Newton: five iterations, secant: eight iterations

461. $E = 4.071$

463. 4.394%

465. $F'(x) = 15x^2 + 4x + 3$

467. $F'(x) = 2xe^x + x^2 e^x$

469. $F'(x) = e^x$

471. $F(x) = e^x - x^3 - \cos(x) + C$

473. $F(x) = \frac{x^2}{2} - x - 2\cos(2x) + C$

475. $F(x) = \frac{1}{2}x^2 + 4x^3 + C$

477. $F(x) = \frac{2}{5}(\sqrt{x})^5 + C$

479. $F(x) = \frac{3}{2}x^{2/3} + C$

481. $F(x) = x + \tan(x) + C$

483. $F(x) = \frac{1}{3}\sin^3(x) + C$

485. $F(x) = -\frac{1}{2}\cot(x) - \frac{1}{x} + C$

487. $F(x) = -\sec x - 4\csc x + C$

489. $F(x) = -\frac{1}{8}e^{-4x} - \cos x + C$

491. $-\cos x + C$

493. $3x - \frac{2}{x} + C$

495. $\frac{8}{3}x^{3/2} + \frac{4}{5}x^{5/4} + C$

497. $14x - \frac{2}{x} - \frac{1}{2x^2} + C$

499. $f(x) = -\frac{1}{2x^2} + \frac{3}{2}$

501. $f(x) = \sin x + \tan x + 1$

503. $f(x) = -\frac{1}{6}x^3 - \frac{2}{x} + \frac{13}{6}$

505. Answers may vary; one possible answer is $f(x) = e^{-x}$

507. Answers may vary; one possible answer is $f(x) = -\sin x$

509. 5.867 sec

511. 7.333 sec

513. 13.75 ft/sec^2

515. $F(x) = \frac{1}{3}x^3 + 2x$

517. $F(x) = x^2 - \cos x + 1$

519. $F(x) = -\frac{1}{(x+1)} + 1$

521. True
523. False

Review Exercises

525. True, by Mean Value Theorem
527. True
529. Increasing: $(-2, 0) \cup (4, \infty)$, decreasing: $(-\infty, -2) \cup (0, 4)$

531. $L(x) = \frac{17}{16} + \frac{1}{2}(1 + 4\pi)\left(x - \frac{1}{4}\right)$

533. Critical point: $x = \frac{3\pi}{4}$, absolute minimum: $x = 0$, absolute maximum: $x = \pi$

535. Increasing: $(-1, 0) \cup (3, \infty)$, decreasing: $(-\infty, -1) \cup (0, 3)$, concave up: $\left(-\infty, \frac{1}{3}\left(2 - \sqrt{13}\right)\right) \cup \left(\frac{1}{3}\left(2 + \sqrt{13}\right), \infty\right)$, concave down: $\left(\frac{1}{3}\left(2 - \sqrt{13}\right), \frac{1}{3}\left(2 + \sqrt{13}\right)\right)$

537. Increasing: $\left(\frac{1}{4}, \infty\right)$, decreasing: $\left(0, \frac{1}{4}\right)$, concave up: $(0, \infty)$, concave down: nowhere

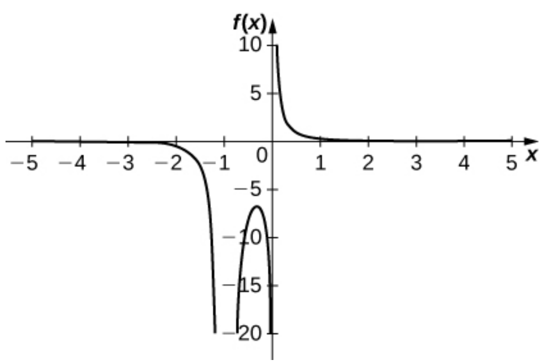
539. 3

541. $-\frac{1}{\pi}$

543. $x_1 = -1, x_2 = -1$

545. $F(x) = \frac{2x^{3/2}}{3} + \frac{1}{x} + C$

547.



Inflection points: none; critical points: $x = -\frac{1}{3}$; zeros: none; vertical asymptotes: $x = -1$, $x = 0$; horizontal asymptote: $y = 0$

549. The height is decreasing at a rate of 0.125 m/sec

551. $x = \sqrt{ab}$ feet

Chapter 5

Checkpoint

5.1. $\sum_{i=3}^{6} 2^i = 2^3 + 2^4 + 2^5 + 2^6 = 120$

5.2. 15,550
5.3. 440
5.4. The left-endpoint approximation is 0.7595. The right-endpoint approximation is 0.6345. See the below **image**.