429. $\frac{1}{2}\ln\left(\frac{4}{3}\right)$

431. $1-\frac{2}{\sqrt{5}}$

433. $2\tan^{-1}(A)\to\pi$ as $A\to\infty$

435. Using the hint, one has $\int\frac{\csc^2 x}{\csc^2 x+\cot^2 x}dx=\int\frac{\csc^2 x}{1+2\cot^2 x}dx$. Set $u=\sqrt{2}\cot x$. Then, $du=-\sqrt{2}\csc^2 x$ and the integral is $-\frac{1}{\sqrt{2}}\int\frac{du}{1+u^2}=-\frac{1}{\sqrt{2}}\tan^{-1}u+C=\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\cot x\right)+C$. If one uses the identity $\tan^{-1}s+\tan^{-1}\left(\frac{1}{s}\right)=\frac{\pi}{2}$, then this can also be written $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$.

437. $x\approx\pm 1.7321$. The left endpoint estimate with $N=100$ is 4.781 and these decimals persist for $N=500$.

Review Exercises

439. False

441. True

443. $L_4=5.25,\ R_4=3.25,$ exact answer: 4

445. $L_4=5.364,\ R_4=5.364,$ exact answer: 5.870

447. $-\frac{4}{3}$

449. 1

451. $-\frac{1}{2(x+4)^2}+C$

453. $\frac{4}{3}\sin^{-1}\left(x^3\right)+C$

455. $\frac{\sin t}{\sqrt{1+t^2}}$

457. $4\frac{\ln x}{x}+1$

459. \$6,328,113

461. \$73.36

463. $\frac{19117}{12}$ft/sec, or 1593 ft/sec

Chapter 6

Checkpoint

6.1. 12 units^2

6.2. $\frac{3}{10}$ unit^2

6.3. $2+2\sqrt{2}$ units^2

6.4. $\frac{5}{3}$ units^2

6.5. $\frac{5}{3}$ units^2

6.7. $\frac{\pi}{2}$

6.8. 8π units^3

6.9. 21π units^3

6.10. $\frac{10\pi}{3}$ units^3

6.11. 60π units^3

6.12. $\frac{15\pi}{2}$ units^3

6.13. 8π units^3

6.14. 12π units^3

6.15. $\frac{11\pi}{6}$ units^3

6.16. $\frac{\pi}{6}$ units^3

6.17. Use the method of washers; $V = \int_{-1}^{1} \pi\left[\left(2 - x^2\right)^2 - \left(x^2\right)^2\right]dx$

6.18. $\frac{1}{6}\left(5\sqrt{5} - 1\right) \approx 1.697$

6.19. Arc Length ≈ 3.8202

6.20. Arc Length $= 3.15018$

6.21. $\frac{\pi}{6}\left(5\sqrt{5} - 3\sqrt{3}\right) \approx 3.133$

6.22. 12π

6.23. 70/3

6.24. 24π

6.25. 8 ft-lb

6.26. Approximately 43,255.2 ft-lb

6.27. 156,800 N

6.28. Approximately 7,164,520,000 lb or 3,582,260 t

6.29. $M = 24,\ \bar{x} = \frac{2}{5}$ m

6.30. $(-1, -1)$ m

6.31. The centroid of the region is $(3/2, 6/5)$.

6.32. The centroid of the region is $(1, 13/5)$.

6.33. The centroid of the region is $(0, 2/5)$.

6.34. $6\pi^2$ units^3

6.35.

a. $\frac{d}{dx}\ln\left(2x^2 + x\right) = \frac{4x + 1}{2x^2 + x}$

b. $\frac{d}{dx}\left(\ln\left(x^3\right)\right)^2 = \frac{6\ln\left(x^3\right)}{x}$

6.36. $\int \frac{x^2}{x^3 + 6}dx = \frac{1}{3}\ln\left|x^3 + 6\right| + C$

6.37. $4 \ln 2$

6.38.

a. $\frac{d}{dx}\left(\frac{e^{x^2}}{e^{5x}}\right) = e^{x^2 - 5x}(2x - 5)$

b. $\frac{d}{dt}\left(e^{2t}\right)^3 = 6e^{6t}$

6.39. $\int \frac{4}{e^{3x}}dx = -\frac{4}{3}e^{-3x} + C$

6.40.

a. $\frac{d}{dt}4^{t^4} = 4^{t^4}(\ln 4)\left(4t^3\right)$

b. $\frac{d}{dx}\log_3\left(\sqrt{x^2 + 1}\right) = \frac{x}{(\ln 3)\left(x^2 + 1\right)}$

6.41. $\int x^2 2^{x^3}dx = \frac{1}{3\ln 2}2^{x^3} + C$

6.42. There are 81,377,396 bacteria in the population after 4 hours. The population reaches 100 million bacteria after

244.12 minutes.

6.43. At 5% interest, she must invest $223,130.16. At 6% interest, she must invest $165,298.89.

6.44. 38.90 months

6.45. The coffee is first cool enough to serve about 3.5 minutes after it is poured. The coffee is too cold to serve about 7 minutes after it is poured.

6.46. A total of 94.13 g of carbon remains. The artifact is approximately 13,300 years old.

6.47.

a. $\frac{d}{dx}\left(\tanh\left(x^2+3x\right)\right)=\left(\operatorname{sech}^2\left(x^2+3x\right)\right)(2x+3)$

b. $\frac{d}{dx}\left(\frac{1}{(\sinh x)^2}\right)=\frac{d}{dx}(\sinh x)^{-2}=-2(\sinh x)^{-3}\cosh x$

6.48.

a. $\int \sinh^3 x\cosh x\,dx=\frac{\sinh^4 x}{4}+C$

b. $\int \operatorname{sech}^2(3x)dx=\frac{\tanh(3x)}{3}+C$

6.49.

a. $\frac{d}{dx}\left(\cosh^{-1}(3x)\right)=\frac{3}{\sqrt{9x^2-1}}$

b. $\frac{d}{dx}\left(\coth^{-1}x\right)^3=\frac{3\left(\coth^{-1}x\right)^2}{1-x^2}$

6.50.

a. $\int \frac{1}{\sqrt{x^2-4}}dx=\cosh^{-1}\left(\frac{x}{2}\right)+C$

b. $\int \frac{1}{\sqrt{1-e^{2x}}}dx=-\operatorname{sech}^{-1}(e^x)+C$

6.51. 52.95 ft

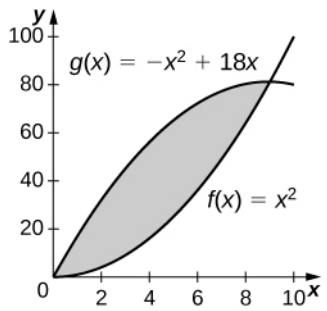
Section Exercises

1. $\frac{32}{3}$

3. $\frac{13}{12}$

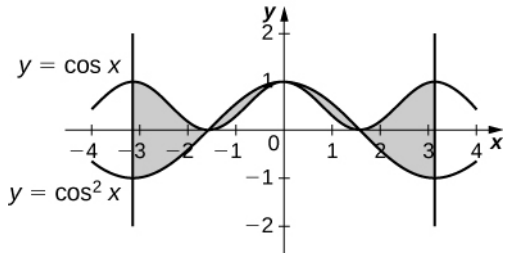
5. 36

7.



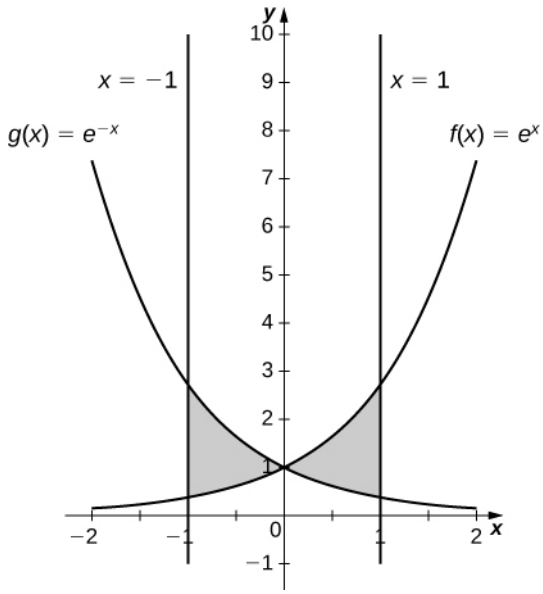
243 square units

9.



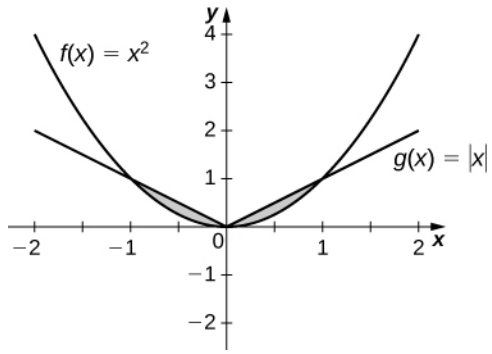
4

11.



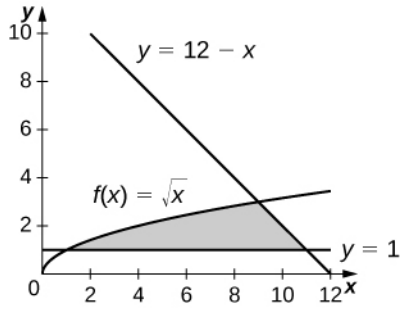
$\frac{2(e-1)^2}{e}$

13.



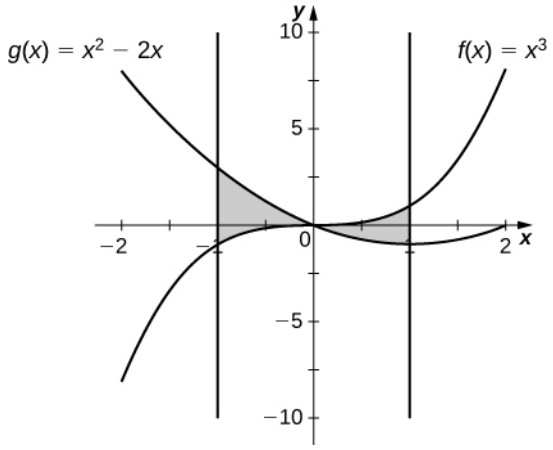
$\frac{1}{3}$

15.



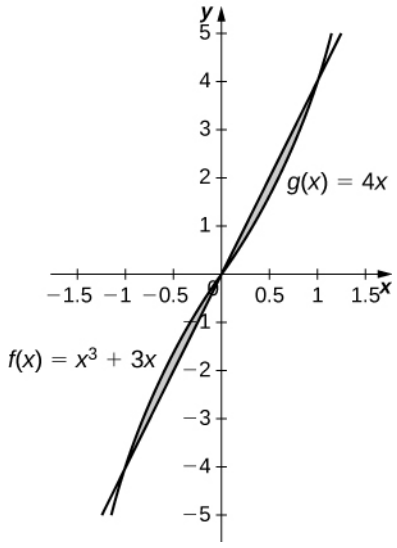
$\frac{34}{3}$

17.



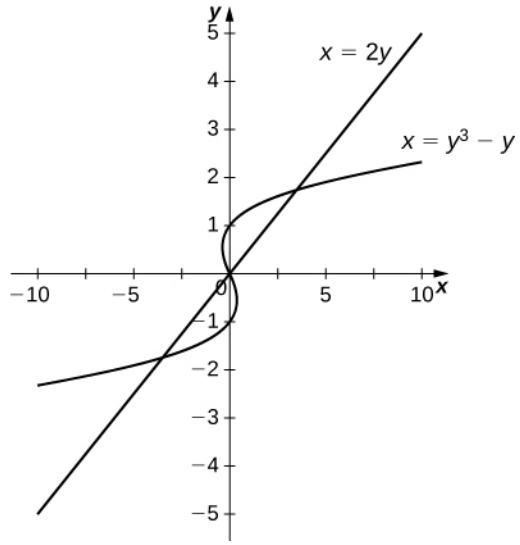
$\frac{5}{2}$

19.



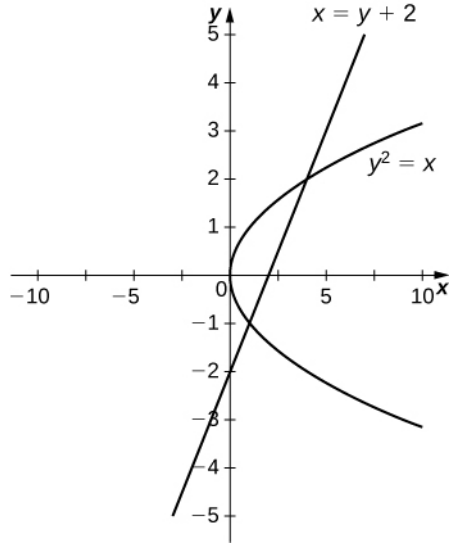
$\frac{1}{2}$

21.



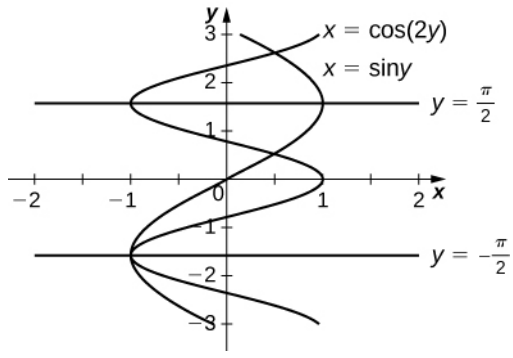
$\frac{9}{2}$

23.



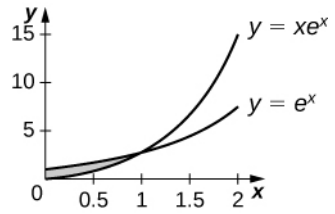
$\frac{9}{2}$

25.



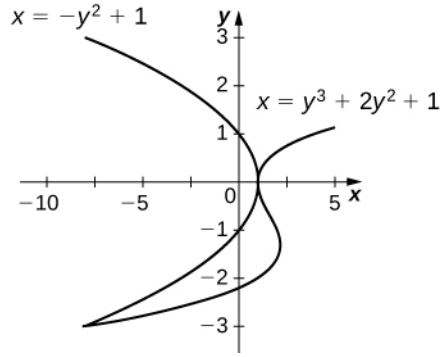
$\frac{3\sqrt{3}}{2}$

27.



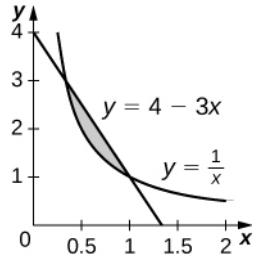
$e-2$

29.



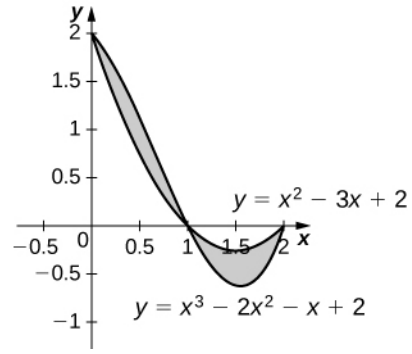
$\frac{27}{4}$

31.



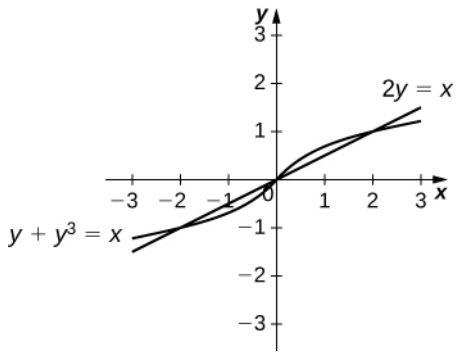
$\frac{4}{3} - \ln(3)$

33.



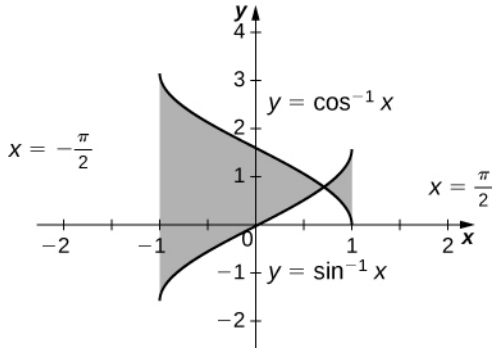
$\frac{1}{2}$

35.



$\frac{1}{2}$

37.



$-2\left(\sqrt{2}-\pi\right)$

39. 1.067

41. 0.852

43. 7.523

45. $\frac{3\pi-4}{12}$

47. 1.429

49. \$33,333.33 total profit for 200 cell phones sold

51. 3.263 mi represents how far ahead the hare is from the tortoise

53. $\frac{343}{24}$

55. $4\sqrt{3}$

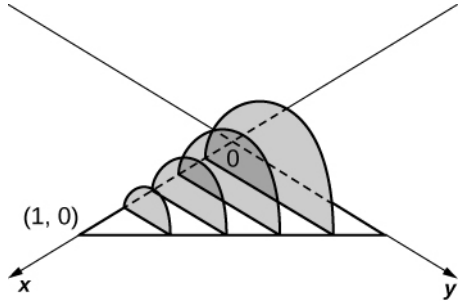
57. $\pi-\frac{32}{25}$

63. 8 units^3

65. $\frac{32}{3\sqrt{2}}$ units^3

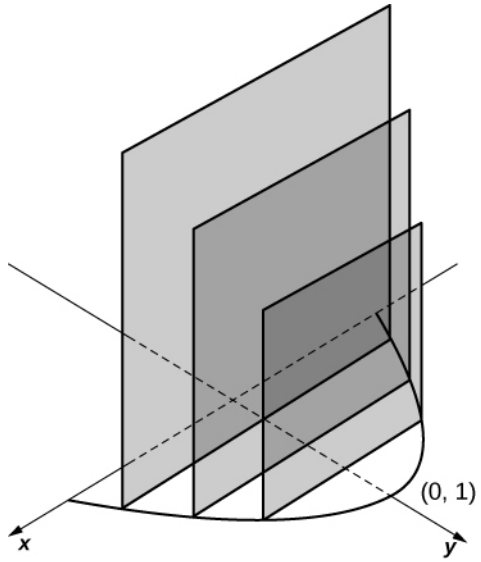
67. $\frac{7\pi}{12}hr^2$ units^3

69.



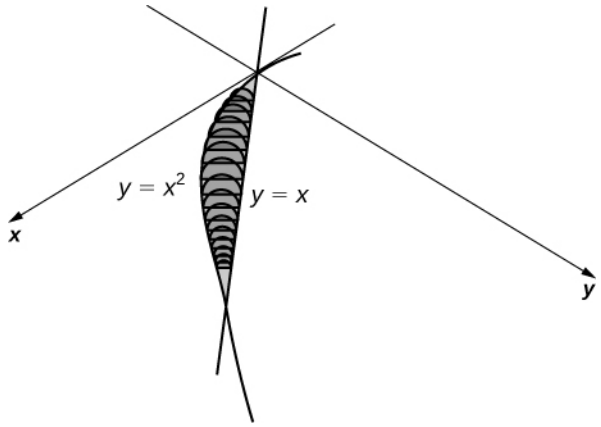
$\frac{\pi}{24}$ units^3

71.



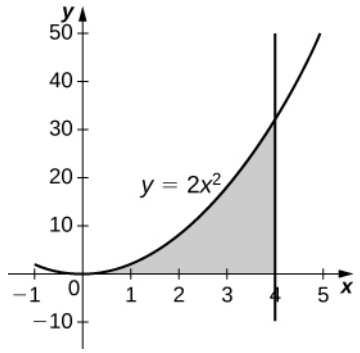
2 units^3

73.



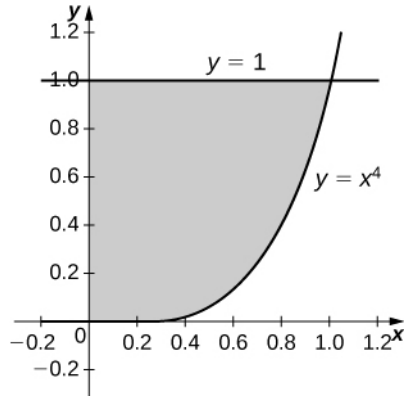
$\frac{\pi}{240}$ units^3

75.



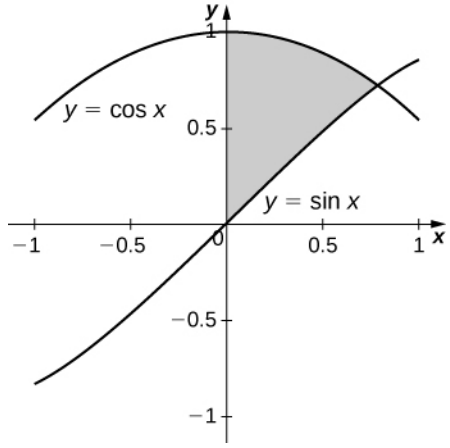
$\frac{4096\pi}{5}$ units^3

77.



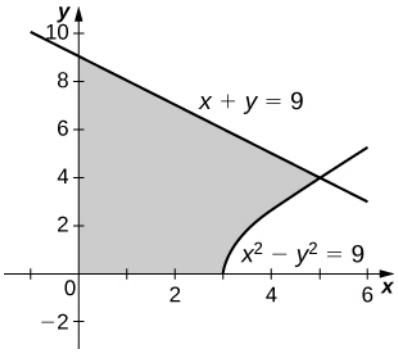
$\frac{8\pi}{9}$ units^3

79.



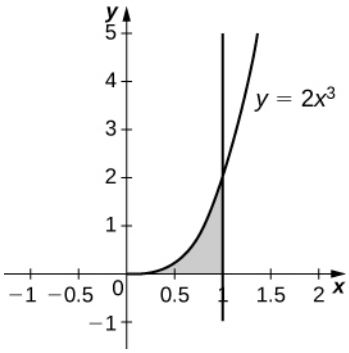
$\frac{\pi}{2}$ units^3

81.



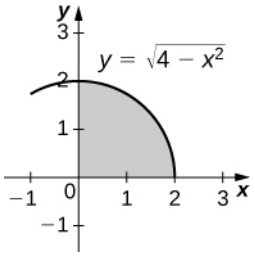
207π units^3

83.



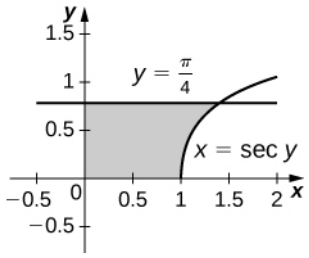
$\frac{4\pi}{5}$ units^3

85.



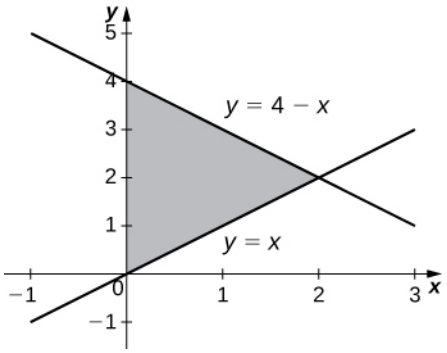
$\frac{16\pi}{3}$ units^3

87.



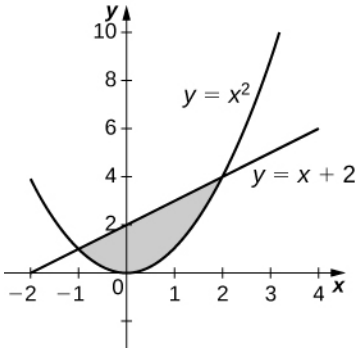
π units^3

89.



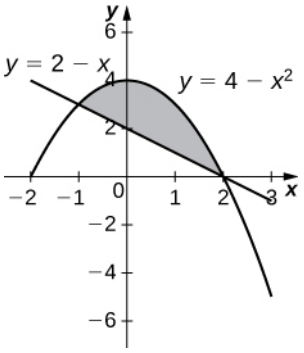
$\frac{16\pi}{3}$ units^3

91.



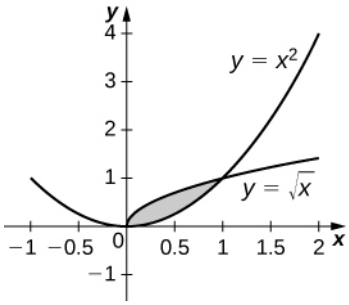
$\frac{72\pi}{5}$ units^3

93.



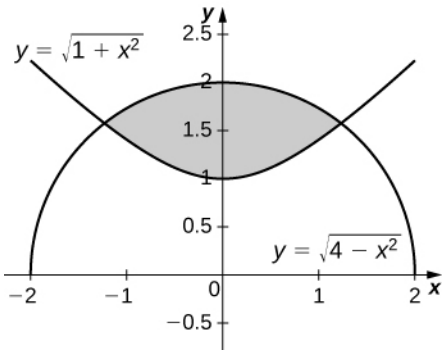
$\frac{108\pi}{5}$ units^3

95.



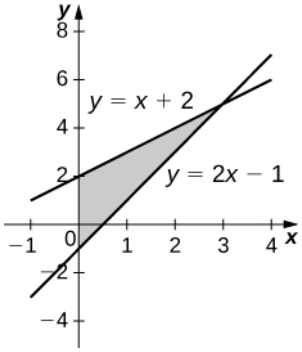
$\frac{3\pi}{10}$ units^3

97.



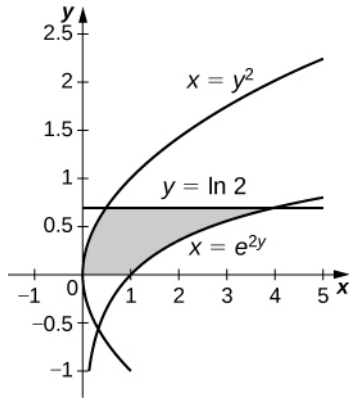
$2\sqrt{6}\pi$ units^3

99.



9π units^3

101.



$\frac{\pi}{20}\left(75 - 4\ln^5(2)\right)$ units^3

103. $\frac{m^2\pi}{3}\left(b^3 - a^3\right)$ units^3

105. $\frac{4a^2b\pi}{3}$ units^3

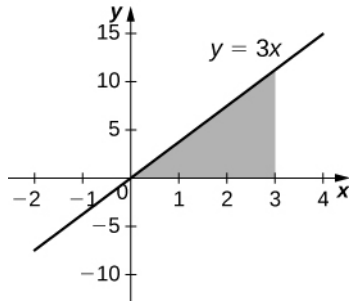
107. $2\pi^2$ units^3

109. $\frac{2ab^2\pi}{3}$ units^3

111. $\frac{\pi}{12}(r+h)^2(6r-h)$ units^3

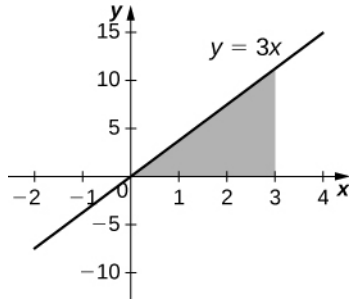
113. $\frac{\pi}{3}(h+R)(h-2R)^2$ units^3

115.



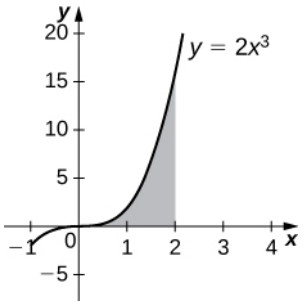
54π units^3

117.



81π units^3

119.



$\frac{512\pi}{7}$ units^3

121. 2π units^3

123. $\frac{2\pi}{3}$ units^3

125. 2π units^3

127. $\frac{4\pi}{5}$ units^3

129. $\frac{64\pi}{3}$ units^3

131. $\frac{32\pi}{5}$ units^3

133. $\frac{7\pi}{6}$

135. 48π

137. $\frac{97\pi}{5}$

139. $\frac{512\pi}{7}$

141. $\frac{64\pi}{5}$ units^3

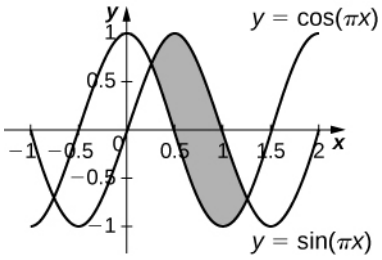
143. $\frac{28\pi}{15}$ units^3

145. $\frac{3\pi}{10}$ units^3

147. $\frac{52\pi}{5}$ units^3

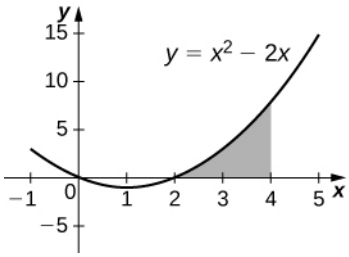
149. 0.9876 units^3

151.



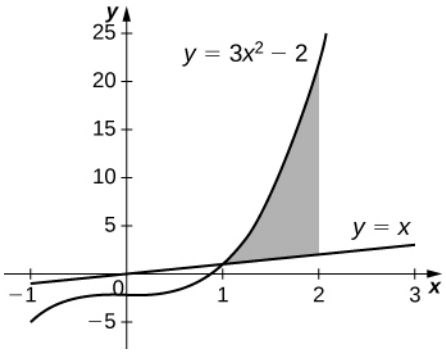
$3\sqrt{2}$ units^3

153.



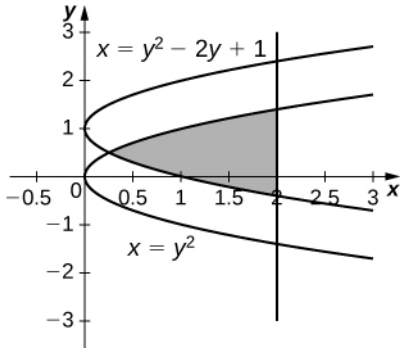
$\frac{496\pi}{15}$ units^3

155.



$\frac{398\pi}{15}$ units^3

157.



15.9074 units^3

159. $\frac{1}{3}\pi r^2 h$ units^3

161. $\pi r^2 h$ units^3

163. πa^2 units^3

165. $2\sqrt{26}$

167. $2\sqrt{17}$

169. $\frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)$

171. $\frac{13\sqrt{13} - 8}{27}$

173. $\frac{4}{3}$

175. 2.0035

177. $\frac{123}{32}$

179. 10

181. $\frac{20}{3}$

183. $\frac{1}{675}\left(229\sqrt{229}-8\right)$

185. $\frac{1}{8}\left(4\sqrt{5}+\ln\left(9+4\sqrt{5}\right)\right)$

187. 1.201

189. 15.2341

191. $\frac{49\pi}{3}$

193. $70\pi\sqrt{2}$

195. 8π

197. $120\pi\sqrt{26}$

199. $\frac{\pi}{6}(17\sqrt{17}-1)$

201. $9\sqrt{2}\pi$

203. $\frac{10\sqrt{10}\pi}{27}\left(73\sqrt{73}-1\right)$

205. 25.645

207. 2π

209. 10.5017

211. 23 ft

213. 2

215. Answers may vary

217. For more information, look up Gabriel's Horn.

219. 150 ft-lb

221. 200 J

223. 1 J

225. $\frac{39}{2}$

227. ln(243)

229. $\frac{332\pi}{15}$

231. 100π

233. $20\pi\sqrt{15}$

235. 6 J

237. 5 cm

239. 36 J

241. 18,750 ft-lb

243. $\frac{32}{3}\times 10^9$ ft-lb

245. 9.71×10^2 N m

247. a. 3,000,000 lb, b. 749,000 lb

249. 23.25π million ft-lb

251. $\frac{A\rho H^2}{2}$

253. Answers may vary

255. $\frac{5}{4}$

257. $\left(\frac{2}{3}, \frac{2}{3}\right)$

259. $\left(\frac{7}{4}, \frac{3}{2}\right)$

261. $\frac{3L}{4}$

263. $\frac{\pi}{2}$

265. $\frac{e^2+1}{e^2-1}$

267. $\frac{\pi^2-4}{\pi}$

269. $\frac{1}{4}\left(1+e^2\right)$

271. $\left(\frac{a}{3}, \frac{b}{3}\right)$

273. $\left(0, \frac{\pi}{8}\right)$

275. $(0,\ 3)$

277. $\left(0, \frac{4}{\pi}\right)$

279. $\left(\frac{5}{8}, \frac{1}{3}\right)$

281. $\frac{m\pi}{3}$

283. $\pi a^2 b$

285. $\left(\frac{4}{3\pi}, \frac{4}{3\pi}\right)$

287. $\left(\frac{1}{2}, \frac{2}{5}\right)$

289. $\left(0, \frac{28}{9\pi}\right)$

291. Center of mass: $\left(\frac{a}{6}, \frac{4a^2}{5}\right)$, volume: $\frac{2\pi a^4}{9}$

293. Volume: $2\pi^2 a^2 (b+a)$

295. $\frac{1}{x}$

297. $-\frac{1}{x(\ln x)^2}$

299. $\ln(x+1)+C$

301. $\ln(x)+1$

303. $\cot(x)$

305. $\frac{7}{x}$

307. $\csc(x)\sec x$

309. $-2\tan x$

311. $\frac{1}{2}\ln\left(\frac{5}{3}\right)$

313. $2-\frac{1}{2}\ln(5)$

315. $\frac{1}{\ln(2)}-1$

317. $\frac{1}{2}\ln(2)$

319. $\frac{1}{3}(\ln x)^3$

321. $\frac{2x^3}{\sqrt{x^2+1}\sqrt{x^2-1}}$

323. $x^{-2-(1/x)}(\ln x - 1)$

325. ex^{e-1}

327. 1

329. $-\frac{1}{x^2}$

331. $\pi - \ln(2)$

333. $\frac{1}{x}$

335. $e^5 - 6 \text{ units}^2$

337. $\ln(4) - 1 \text{ units}^2$

339. 2.8656

341. 3.1502

349. True

351. False; $k = \frac{\ln(2)}{t}$

353. 20 hours

355. No. The relic is approximately 871 years old.

357. 71.92 years

359. 5 days 6 hours 27 minutes

361. 12

363. 8.618%

365. \$6766.76

367. 9 hours 13 minutes

369. 239,179 years

371. $P'(t) = 43e^{0.01604t}$. The population is always increasing.

373. The population reaches 10 billion people in 2027.

375. $P'(t) = 2.259e^{0.06407t}$. The population is always increasing.

377. e^x and e^{-x}

379. Answers may vary

381. Answers may vary

383. Answers may vary

385. $3\sinh(3x+1)$

387. $-\tanh(x)\text{sech}(x)$

389. $4\cosh(x)\sinh(x)$

391. $\frac{x\,\text{sech}^2\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$

393. $6\sinh^5(x)\cosh(x)$

395. $\frac{1}{2}\sinh(2x+1) + C$

397. $\frac{1}{2}\sinh^2\left(x^2\right) + C$

399. $\frac{1}{3}\cosh^3(x) + C$

401. $\ln(1+\cosh(x)) + C$

403. $\cosh(x) + \sinh(x) + C$

405. $\frac{4}{1-16x^2}$

407. $\frac{\sinh(x)}{\sqrt{\cosh^2(x)+1}}$

409. $-\csc(x)$

411. $-\frac{1}{\left(x^2-1\right)\tanh^{-1}(x)}$

413. $\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C$

415. $\sqrt{x^2+1}+C$

417. $\cosh^{-1}(e^x)+C$

419. Answers may vary

421. 37.30

423. $y=\frac{1}{c}\cosh(cx)$

425. -0.521095

427. 10

Review Exercises

435. False

437. False

439. $32\sqrt{3}$

441. $\frac{162\pi}{5}$

443. a. 4, b. $\frac{128\pi}{7}$, c. $\frac{64\pi}{5}$

445. a. 1.949, b. 21.952, c. 17.099

447. a. $\frac{31}{6}$, b. $\frac{452\pi}{15}$, c. $\frac{31\pi}{6}$

449. 245.282

451. Mass: $\frac{1}{2}$, center of mass: $\left(\frac{18}{35},\frac{9}{11}\right)$

453. $\sqrt{17}+\frac{1}{8}\ln(33+8\sqrt{17})$

455. Volume: $\frac{3\pi}{4}$, surface area: $\pi\left(\sqrt{2}-\sinh^{-1}(1)+\sinh^{-1}(16)-\frac{\sqrt{257}}{16}\right)$

457. 11:02 a.m.

459. $\pi(1+\sinh(1)\cosh(1))$