213. 5 **215**. 8/7 217. DNE **219**. 2/3 **221**. -4; **223.** Since $-1 \le \cos(2\pi x) \le 1$, then $-x^2 \le x^2 \cos(2\pi x) \le x^2$. Since $\lim_{x \to 0} x^2 = 0 = \lim_{x \to 0} -x^2$, it follows that $\lim_{x \to 0} x^2 \cos(2\pi x) = 0.$ **225**. [2, ∞] **227**. c = -1**229**. $\delta = \sqrt[3]{\varepsilon}$ **231**. 0 m/sec Chapter 3 Checkpoint **3.1**. $\frac{1}{4}$ **3.2**. 6 **3.3**. f'(1) = 5**3.4**. -32 ft/s **3.5**. P'(3.25) = 20 > 0; raise prices **3.6**. f'(x) = 2x**3.7**. $(0, +\infty)$ **3.8**. a = 6 and b = -9**3.9**. f''(x) = 2**3.10**. a(t) = 6t**3.11**. 0 **3.12**. $4x^3$ **3.13**. $f'(x) = 7x^6$ **3.14**. $f'(x) = 6x^2 - 12x$. **3.15**. y = 12x - 23**3.16.** $j'(x) = 10x^4(4x^2 + x) + (8x + 1)(2x^5) = 56x^6 + 12x^5.$ **3.17**. $k'(x) = -\frac{13}{(4x-3)^2}$. **3.18**. $g'(x) = -7x^{-8}$. **3.19**. 3f'(x) - 2g'(x). **3.20**. $\frac{5}{8}$ **3.21**. -4.4 3.22. left to right 3.23. 3,300 3.24. \$2 **3.25**. $f'(x) = \cos^2 x - \sin^2 x$ **3.26**. $\frac{\cos x + x \sin x}{\cos^2 x}$ **3.27**. $t = \frac{\pi}{3}, t = \frac{2\pi}{3}$ **3.28**. $f'(x) = -\csc^2 x$

3.29. $f'(x) = 2\sec^2 x + 3\csc^2 x$ **3.30**. $\frac{4}{3}$ **3.31**. cos x **3.32**. $-\cos x$ **3.33**. $v\left(\frac{5\pi}{6}\right) = -\sqrt{3} < 0$ and $a\left(\frac{5\pi}{6}\right) = -1 < 0$. The block is speeding up. **3.34.** $h'(x) = 4(2x^3 + 2x - 1)^3(6x^2 + 2) = 8(3x^2 + 1)(2x^3 + 2x - 1)^3$ **3.35**. y = -48x - 88**3.36**. $h'(x) = 7\cos(7x + 2)$ **3.37**. $h'(x) = \frac{3-4x}{(2x+3)^4}$ **3.38**. $h'(x) = 18x^2 \sin^5(x^3) \cos(x^3)$ **3.39**. $a(t) = -16\sin(4t)$ **3.40**. 28 **3.41**. $\frac{dy}{dx} = -3x^2 \sin(x^3)$ **3.42**. $g'(x) = -\frac{1}{(x+2)^2}$ **3.43**. $g(x) = \frac{1}{5}x^{-4/5}$ **3.44**. $s'(t) = (2t+1)^{-1/2}$ **3.45**. $g'(x) = \frac{1}{1+x^2}$ **3.46**. $h'(x) = \frac{-3}{\sqrt{6x - 9x^2}}$ **3.47**. y = x**3.48**. $\frac{dy}{dx} = \frac{5 - 20x^4}{\sec^2 v - 2v}$ **3.49**. $y = \frac{5}{3}x - \frac{16}{3}$ **3.50**. $h'(x) = e^{2x} + 2xe^{2x}$ **3.51**. 996 **3.52.** $f'(x) = \frac{15}{3x+2}$ **3.53**. 9ln(3) **3.54**. $\frac{dy}{dx} = x^{x}(1 + \ln x)$ **3.55**. $y' = \pi (\tan x)^{\pi - 1} \sec^2 x$ Section Exercises **1**. 4 **3**. 8.5 **5**. $-\frac{3}{4}$ 7. 0.2 9. 0.25 **11**. a. -4 b. y = 3 - 4x

13. a. 3 b. y = 3x - 1

15. a. $\frac{-7}{9}$ b. $y = \frac{-7}{9}x + \frac{14}{3}$ **17**. a. 12 b. y = 12x + 14**19**. a. -2 b. y = -2x - 10**21**. 5 **23**. 13 **25**. $\frac{1}{4}$ **27**. $-\frac{1}{4}$ **29**. –3 (i) 5.100000, (iv) 5.000100, (v) 5.000010, (ii) 5.010000, (iii) 5.001000, **31**. a. (vi) 5.000001, (vii) 4.900000, (viii) 4.990000, (ix) 4.999000, (x) 4.999900, (xi) 4.999990, (x) 4.999999 b. $m_{tan} = 5$ c. y = 5x + 3**33**. a. (i) 4.8771, (ii) 4.9875 (iii) 4.9988, (iv) 4.9999, (v) 4.9999, (vi) 4.9999 b. $m_{tan} = 5$ c. y = 5x + 10**35.** a. $\frac{1}{3}$; b. (i) 0. $\overline{3}$ m/s, (ii) 0. $\overline{3}$ m/s, (iii) 0. $\overline{3}$ m/s, (iv) 0. $\overline{3}$ m/s; c. 0. $\overline{3} = \frac{1}{3}$ m/s **37**. a. $2(h^2 + 6h + 12)$; b. (i) 25.22 m/s, (ii) 24.12 m/s, (iii) 24.01 m/s, (iv) 24 m/s; c. 24 m/s **39**. a. 1.25; b. 0.5 **41.** $\lim_{x \to 0^{-}} \frac{x^{1/3} - 0}{x - 0} = \lim_{x \to 0^{-}} \frac{1}{x^{2/3}} = \infty$ **43.** $\lim_{x \to 1^{-}} \frac{1-1}{x-1} = 0 \neq 1 = \lim_{x \to 1^{+}} \frac{x-1}{x-1}$ **45**. a. (i) 61.7244 ft/s, (ii) 61.0725 ft/s (iii) 61.0072 ft/s (iv) 61.0007 ft/s b. At 4 seconds the race car is traveling at a

45. a. (i) 61.7244 ft/s, (ii) 61.0725 ft/s (iii) 61.0072 ft/s (iv) 61.0007 ft/s b. At 4 seconds the race car is traveling at a rate/velocity of 61 ft/s.

47. a. The vehicle represented by f(t), because it has traveled 2 feet, whereas g(t) has traveled 1 foot. b. The velocity of f(t) is constant at 1 ft/s, while the velocity of g(t) is approximately 2 ft/s. c. The vehicle represented by g(t), with a velocity of approximately 4 ft/s. d. Both have traveled 4 feet in 4 seconds. **49**. a.



b. $a \approx -1.361, 2.694$

51. a. $N(x) = \frac{x}{30}$ b. ~ 3.3 gallons. When the vehicle travels 100 miles, it has used 3.3 gallons of gas. c. $\frac{1}{30}$. The rate of gas consumption in gallons per mile that the vehicle is achieving after having traveled 100 miles. **53.** a.







77. a.



89. $f'(x) = 3x^2$

f'(x) 20 f'(x) 20 10 -4 -2 0 2 x -10f(x) -20

91. a. Average rate at which customers spent on concessions in thousands per customer. b. Rate (in thousands per customer) at which x customers spent money on concessions in thousands per customer.

93. a. Average grade received on the test with an average study time between two values. b. Rate (in percentage points per hour) at which the grade on the test increased or decreased for a given average study time of x hours.

95. a. Average change of atmospheric pressure between two different altitudes. b. Rate (torr per foot) at which atmospheric pressure is increasing or decreasing at x feet.

97. a. The rate (in degrees per foot) at which temperature is increasing or decreasing for a given height *x*. b. The rate of change of temperature as altitude changes at 1000 feet is -0.1 degrees per foot.

99. a. The rate at which the number of people who have come down with the flu is changing t weeks after the initial outbreak. b. The rate is increasing sharply up to the third week, at which point it slows down and then becomes constant. **101**.

Time (seconds)	<i>h'</i> (<i>t</i>) (m/s)
0	2
1	2
2	5.5
3	10.5
4	9.5
5	7

103. G'(t) = 2.858t + 0.0857



105. H''(t) = 0, G''(t) = 2.858 and f''(t) = 1.222t + 5.912 represent the acceleration of the rocket, with units of meters per second squared (m/s²).

107. $f'(x) = 15x^2 - 1$ **109.** $f'(x) = 32x^3 + 18x$ **111.** $f'(x) = 270x^4 + \frac{39}{(x+1)^2}$ **113.** $f'(x) = \frac{-5}{x^2}$ **115.** $f'(x) = \frac{4x^4 + 2x^2 - 2x}{x^4}$ **117.** $f'(x) = \frac{-x^2 - 18x + 64}{(x^2 - 7x + 1)^2}$ **119.**





$$T(x) = 4x - 5$$

123. $h'(x) = 3x^2 f(x) + x^3 f'(x)$
125. $h'(x) = \frac{3f'(x)(g(x) + 2) - 3f(x)g'(x)}{(g(x) + 2)^2}$

129. Undefined
131. a. 2, b. does not exist, c. 2.5
133. a. 23, b. y = 23x - 28



135. a. 3, b. y = 3x + 2



c. Acceleration is positive on (5, 7), negative on (0, 2), and zero on (2, 5). d. The object is speeding up on $(6, 7) \cup (1.5, 2)$ and slowing down on $(0, 1.5) \cup (5, 6)$.

161. a. $R(x) = 10x - 0.001x^2$ b. R'(x) = 10 - 0.002x c. \$6 per item, \$0 per item

163. a. C'(x) = 65 b. $R(x) = 143x - 0.03x^2$, R'(x) = 143 - 0.06x c. 83, -97. At a production level of 1000 cordless drills, revenue is increasing at a rate of \$83 per drill; at a production level of 4000 cordless drills, revenue is decreasing at a rate of \$97 per drill. d. $P(x) = -0.03x^2 + 78x - 75000$, P'(x) = -0.06x + 78 e. 18, -162. At a production level of 1000 cordless drills, profit is increasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$18 per drill; at a production level of 4000 cordless drills, profit is decreasing at a rate of \$162 per drill.

165. a.
$$N'(t) = 3000 \left(\frac{-4t^2 + 400}{(t^2 + 100)^2} \right)$$
 b. 120, 0, -14.4, -9.6 c. The bacteria population increases from time 0 to 10 hours;

afterwards, the bacteria population decreases. d. 0, -6, 0.384, 0.432. The rate at which the bacteria is increasing is decreasing during the first 10 hours. Afterwards, the bacteria population is decreasing at a decreasing rate.

167. a. P(t) = 0.03983 + 0.4280 b. P'(t) = 0.03983. The population is increasing. c. P''(t) = 0. The rate at which the population is increasing is constant.

169. a. $p(t) = -0.6071x^2 + 0.4357x - 0.3571$ b. p'(t) = -1.214x + 0.4357. This is the velocity of the sensor. c. p''(t) = -1.214. This is the acceleration of the sensor; it is a constant acceleration downward. **171**. a.



b. f'(x) = a. The more increase in prey, the more growth for predators. c. f''(x) = 0. As the amount of prey increases, the rate at which the predator population growth increases is constant. d. This equation assumes that if there is more prey, the predator is able to increase consumption linearly. This assumption is unphysical because we would expect there to be some saturation point at which there is too much prey for the predator to consume adequately. **173.** a.



b.
$$f'(x) = \frac{2axn^2}{(n^2 + x^2)^2}$$
. When the amount of prey increases, the predator growth increases. c. $f''(x) = \frac{2an^2(n^2 - 3x^2)}{(n^2 + x^2)^3}$. When

the amount of prey is extremely small, the rate at which predator growth is increasing is increasing, but when the amount of prey reaches above a certain threshold, the rate at which predator growth is increasing begins to decrease. d. At lower levels of prey, the prey is more easily able to avoid detection by the predator, so fewer prey individuals are consumed, resulting in less predator growth.

175.
$$\frac{dy}{dx} = 2x - \sec x \tan x$$

177.
$$\frac{dy}{dx} = 2x \cot x - x^2 \csc^2 x$$

179.
$$\frac{dy}{dx} = \frac{x \sec x \tan x - \sec x}{x^2}$$

181.
$$\frac{dy}{dx} = (1 - \sin x)(1 - \sin x) - \cos x(x + \cos x)$$

183.
$$\frac{dy}{dx} = \frac{2 \csc^2 x}{(1 + \cot x)^2}$$

185.
$$y = -x$$









191. $3\cos x - x\sin x$

193. $\frac{1}{2}\sin x$

195. $2\csc x(\csc^2 x + \cot^2 x)$

197. $\frac{(2n+1)\pi}{4}$, where *n* is an integer **199**. $\left(\frac{\pi}{4}, 1\right), \left(\frac{3\pi}{4}, -1\right)$ **201**. *a* = 0, *b* = 3 **203.** $y' = 5\cos(x)$, increasing on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$, and $\left(\frac{7\pi}{2}, 12\right)$ **209**. 3 sin x **211**. $5\cos x$ **213**. $720x^7 - 5\tan(x)\sec^3(x) - \tan^3(x)\sec(x)$ **215.** $18u^2 \cdot 7 = 18(7x - 4)^2 \cdot 7$ **217**. $-\sin u \cdot \frac{-1}{8} = -\sin \left(\frac{-x}{8}\right) \cdot \frac{-1}{8}$ **219**. $\frac{8x-24}{2\sqrt{4u+3}} = \frac{4x-12}{\sqrt{4x^2-24x+3}}$ **221.** a. $u = 3x^2 + 1$; b. $18x(3x^2 + 1)^2$ **223.** a. $f(u) = u^7$, $u = \frac{x}{7} + \frac{7}{x}$; b. $7\left(\frac{x}{7} + \frac{7}{x}\right)^6 \cdot \left(\frac{1}{7} - \frac{7}{x^2}\right)$ **225.** a. $f(u) = \csc u$, $u = \pi x + 1$; b. $-\pi \csc(\pi x + 1) \cdot \cot(\pi x + 1)$ **227**. a. $f(u) = -6u^{-3}$, $u = \sin x$, b. $18 \sin^{-4} x \cdot \cos x$ **229.** $\frac{4}{(5-2x)^3}$ **231.** $6(2x^3 - x^2 + 6x + 1)^2(3x^2 - x + 3)$ **233.** $-3(\tan x + \sin x)^{-4} \cdot (\sec^2 x + \cos x)$ **235**. $-7\cos(\cos 7x) \cdot \sin 7x$ **237**. $-12\cot^2(4x+1)\cdot\csc^2(4x+1)$ **239**. $10\frac{3}{4}$ **241**. $y = \frac{-1}{2}x$ **243**. $x = \pm \sqrt{6}$ **245**. 10 **247**. $-\frac{1}{8}$ **249**. -4 **251**. -12 **253.** a. $-\frac{200}{343}$ m/s, b. $\frac{600}{2401}$ m/s², c. The train is slowing down since velocity and acceleration have opposite signs. **255.** a. $C'(x) = 0.0003x^2 - 0.04x + 3$ b. $\frac{dC}{dt} = 100 \cdot (0.0003x^2 - 0.04x + 3)$ c. Approximately \$90,300 per week **257.** a. $\frac{dS}{dt} = -\frac{8\pi r^2}{(t+1)^3}$ b. The volume is decreasing at a rate of $-\frac{\pi}{36}$ ft³/min. **259**. ~2.3 ft/hr 261. a.



b.
$$(f^{-1})'(1) \sim 2$$

263. a.



269.
$$\frac{1}{5}$$

271. $\frac{1}{3}$
273. 1
275. a. 4, b. $y = 4x$
277. a. $-\frac{1}{96}$, b. $y = -\frac{1}{13}x + \frac{18}{13}$
279. $\frac{2x}{\sqrt{1-x^4}}$
281. $\frac{-1}{\sqrt{1-x^2}}$
283. $\frac{3(1 + \tan^{-1}x)^2}{1 + x^2}$
285. $\frac{-1}{(1 + x^2)(\tan^{-1}x)^2}$
287. $\frac{x}{(5 - x^2)\sqrt{4 - x^2}}$

289. -1
291.
$$\frac{1}{2}$$

293. $\frac{1}{10}$

295. a. $v(t) = \frac{1}{1+t^2}$ b. $a(t) = \frac{-2t}{(1+t^2)^2}$ c. (a)0.2, 0.06, 0.03; (b) - 0.16, -0.028, -0.0088 d. The hockey puck is

decelerating/slowing down at 2, 4, and 6 seconds. **297**. -0.0168 radians per foot

299. a. $\frac{d\theta}{dx} = \frac{10}{100 + x^2} - \frac{40}{1600 + x^2}$ b. $\frac{18}{325}, \frac{9}{340}, \frac{42}{4745}, 0$ c. As a person moves farther away from the screen, the viewing angle is increasing, which implies that as he or she moves farther away, his or her screen vision is widening. d. $-\frac{54}{12905}, -\frac{3}{500}, -\frac{198}{29945}, -\frac{9}{1360}$ e. As the person moves beyond 20 feet from the screen, the viewing angle is decreasing. The optimal distance the person should stand for maximizing the viewing angle is 20 feet.

301. $\frac{dy}{dx} = \frac{-2x}{y}$ 303. $\frac{dy}{dx} = \frac{x}{3y} - \frac{y}{2x}$ 305. $\frac{dy}{dx} = \frac{y - \frac{y}{2\sqrt{x+4}}}{\sqrt{x+4} - x}$

307.
$$\frac{dy}{dx} = \frac{y^2 \cos(xy)}{2y - \sin(xy) - xy \cos xy}$$

309. $\frac{dy}{dx} = \frac{-3x^2y - y^3}{x^3 + 3xy^2}$

311.







$$y = 0$$

317. a.
$$y = -x + 2$$
 b. $(3, -1)$

319. a. $(\pm\sqrt{7}, 0)$ b. -2 c. They are parallel since the slope is the same at both intercepts.

321.
$$y = -x + 1$$

323. a. -0.5926 b. When \$81 is spent on labor and \$16 is spent on capital, the amount spent on capital is decreasing by \$0.5926 per \$1 spent on labor.

325. -8

327. -2.67
329.
$$y' = -\frac{1}{\sqrt{1-x^2}}$$

331. $2xe^x + x^2e^x$
333. $e^{x^3\ln x} (3x^2\ln x + x^2)$
335. $\frac{4}{(e^x + e^{-x})^2}$
337. $2^{4x+2} \cdot \ln 2 + 8x$
339. $\pi x^{\pi-1} \cdot \pi^x + x^{\pi} \cdot \pi^x \ln \pi$
341. $\frac{5}{2(5x-7)}$
343. $\frac{\tan x}{\ln 10}$
345. $2^x \cdot \ln 2 \cdot \log_3 7^{x^2-4} + 2^x \cdot \frac{2x\ln 7}{\ln 3}$

347.
$$(\sin 2x)^{4x} [4 \cdot \ln(\sin 2x) + 8x \cdot \cot 2x]$$

349. $x^{\log_2 x} \cdot \frac{2\ln x}{x \ln 2}$
351. $x^{\cot x} \cdot [-\csc^2 x \cdot \ln x + \frac{\cot x}{x}]$
353. $x^{-1/2} (x^2 + 3)^{2/3} (3x - 4)^4 \cdot \left[\frac{-1}{2x} + \frac{4x}{3(x^2 + 3)} + \frac{12}{3x - 4}\right]$
355.



$$y = \frac{-1}{5 + 5\ln 5}x + \left(5 + \frac{1}{5 + 5\ln 5}\right)$$

357. a. $x = e \sim 2.718$ b. $(e, \infty), (0, e)$

359. a. $P = 500,000(1.05)^t$ individuals b. $P'(t) = 24395 \cdot (1.05)^t$ individuals per year c. 39,737 individuals per year **361.** a. At the beginning of 1960 there were 5.3 thousand cases of the disease in New York City. At the beginning of 1963 there were approximately 723 cases of the disease in the United States. b. At the beginning of 1960 the number of cases of the disease was decreasing at rate of -4.611 thousand per year; at the beginning of 1963, the number of cases of the disease was decreasing at a rate of -0.2808 thousand per year.

363. $p = 35741(1.045)^t$

365.

Years since 1790	Ρ"
0	69.25
10	107.5
20	167.0
30	259.4
40	402.8
50	625.5
60	971.4
70	1508.5

Review Exercises

367. False. 369. False 371. $\frac{1}{2\sqrt{x+4}}$ 373. $9x^2 + \frac{8}{x^3}$ 375. $e^{\sin x} \cos x$ 377. $x \sec^2(x) + 2x \cos(x) + \tan(x) - x^2 \sin(x)$ 379. $\frac{1}{4} \left(\frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \right)$ 381. $\cos x \cdot (\ln x + 1) - x \ln(x) \sin x$ 383. $4^x (\ln 4)^2 + 2 \sin x + 4x \cos x - x^2 \sin x$ 385. T = (2 + e)x - 2387.



389. $w'(3) = -\frac{2.9\pi}{6}$. At 3 a.m. the tide is decreasing at a rate of 1.514 ft/hr.

391. -7.5. The wind speed is decreasing at a rate of 7.5 mph/hr

Chapter 4

Checkpoint

4.1. $\frac{1}{72\pi}$ cm/sec, or approximately 0.0044 cm/sec **4.2.** 500 ft/sec **4.3.** $\frac{1}{10}$ rad/sec **4.4.** -0.61 ft/sec **4.5.** $L(x) = 2 + \frac{1}{12}(x - 8)$; 2.00833 **4.6.** $L(x) = -x + \frac{\pi}{2}$ **4.7.** L(x) = 1 + 4x **4.8.** $dy = 2xe^{x^2} dx$ **4.9.** dy = 1.6, $\Delta y = 1.64$ **4.10.** The volume measurement is accurate to within 21.6 cm³. **4.11.** 7.6% **4.12.** $x = -\frac{2}{3}$, x = 1**4.13.** The absolute maximum is 3 and it occurs at x = 4. The absolute maximum is 3 and it occurs at x = 4.

4.13. The absolute maximum is 3 and it occurs at x = 4. The absolute minimum is -1 and it occurs at x = 2. **4.14**. c = 2

4.15. $\frac{5}{2\sqrt{2}}$ sec