6.2 EXERCISES

58. Derive the formula for the volume of a sphere using the slicing method.

59. Use the slicing method to derive the formula for the volume of a cone.

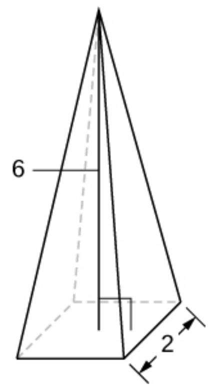
60. Use the slicing method to derive the formula for the volume of a tetrahedron with side length a.

61. Use the disk method to derive the formula for the volume of a trapezoidal cylinder.

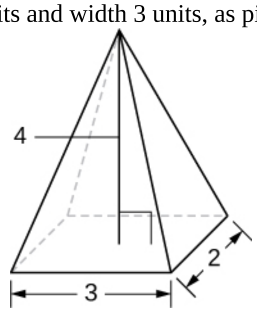
62. Explain when you would use the disk method versus the washer method. When are they interchangeable?

For the following exercises, draw a typical slice and find the volume using the slicing method for the given volume.

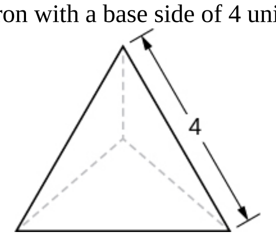
63. A pyramid with height 6 units and square base of side 2 units, as pictured here.



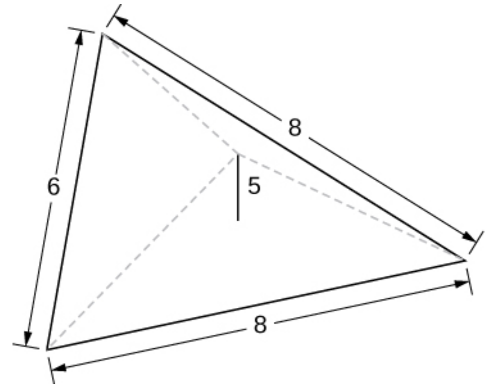
64. A pyramid with height 4 units and a rectangular base with length 2 units and width 3 units, as pictured here.



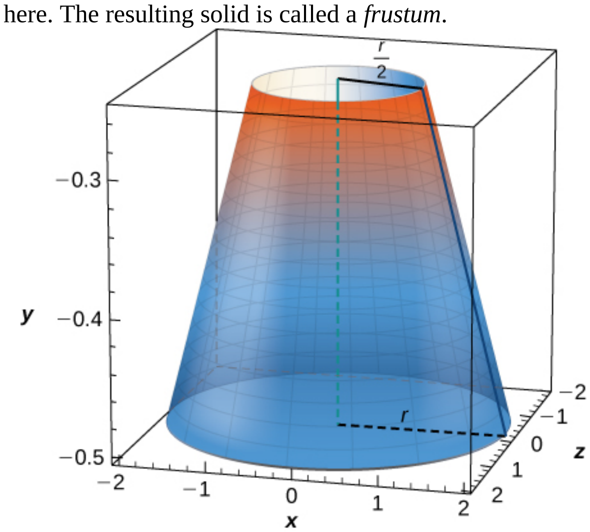
65. A tetrahedron with a base side of 4 units, as seen here.



66. A pyramid with height 5 units, and an isosceles triangular base with lengths of 6 units and 8 units, as seen here.



67. A cone of radius r and height h has a smaller cone of radius $r/2$ and height $h/2$ removed from the top, as seen here. The resulting solid is called a *frustum*.



For the following exercises, draw an outline of the solid and find the volume using the slicing method.

68. The base is a circle of radius a. The slices perpendicular to the base are squares.

69. The base is a triangle with vertices $(0, 0), (1, 0),$ and $(0, 1)$. Slices perpendicular to the x-axis are semicircles.

70. The base is the region under the parabola $y = 1 - x^2$ in the first quadrant. Slices perpendicular to the *xy*-plane are squares.

71. The base is the region under the parabola $y = 1 - x^2$ and above the x-axis. Slices perpendicular to the y-axis are squares.

72. The base is the region enclosed by $y = x^2$ and $y = 9$. Slices perpendicular to the x-axis are right isosceles triangles. The intersection of one of these slices and the base is the leg of the triangle.

73. The base is the area between $y = x$ and $y = x^2$. Slices perpendicular to the x-axis are semicircles.

For the following exercises, draw the region bounded by the curves. Then, use the disk method to find the volume when the region is rotated around the x-axis.

74. $x + y = 8,\ x = 0,$ and $y = 0$

75. $y = 2x^2,\ x = 0,\ x = 4,$ and $y = 0$

76. $y = e^x + 1,\ x = 0,\ x = 1,$ and $y = 0$

77. $y = x^4,\ x = 0,$ and $y = 1$

78. $y = \sqrt{x},\ x = 0,\ x = 4,$ and $y = 0$

79. $y = \sin x,\ y = \cos x,$ and $x = 0$

80. $y = \frac{1}{x},\ x = 2,$ and $y = 3$

81. $x^2 - y^2 = 9$ and $x + y = 9,\ y = 0$ and $x = 0$

For the following exercises, draw the region bounded by the curves. Then, find the volume when the region is rotated around the y-axis.

82. $y = 4 - \frac{1}{2}x,\ x = 0,$ and $y = 0$

83. $y = 2x^3,\ x = 0,\ x = 1,$ and $y = 0$

84. $y = 3x^2,\ x = 0,$ and $y = 3$

85. $y = \sqrt{4 - x^2},\ y = 0,$ and $x = 0$

86. $y = \dfrac{1}{\sqrt{x + 1}},\ x = 0,$ and $x = 3$

87. $x = \sec(y)$ and $y = \frac{\pi}{4},\ y = 0$ and $x = 0$

88. $y = \dfrac{1}{x + 1},\ x = 0,$ and $x = 2$

89. $y = 4 - x,\ y = x,$ and $x = 0$

For the following exercises, draw the region bounded by the curves. Then, find the volume when the region is rotated around the x-axis.

90. $y = x + 2,\ y = x + 6,\ x = 0,$ and $x = 5$

91. $y = x^2$ and $y = x + 2$

92. $x^2 = y^3$ and $x^3 = y^2$

93. $y = 4 - x^2$ and $y = 2 - x$

94. **[T]** $y = \cos x,\ y = e^{-x},\ x = 0,$ and $x = 1.2927$

95. $y = \sqrt{x}$ and $y = x^2$

96. $y = \sin x,\ y = 5\sin x,\ x = 0$ and $x = \pi$

97. $y = \sqrt{1 + x^2}$ and $y = \sqrt{4 - x^2}$

For the following exercises, draw the region bounded by the curves. Then, use the washer method to find the volume when the region is revolved around the y-axis.

98. $y = \sqrt{x},\ x = 4,$ and $y = 0$

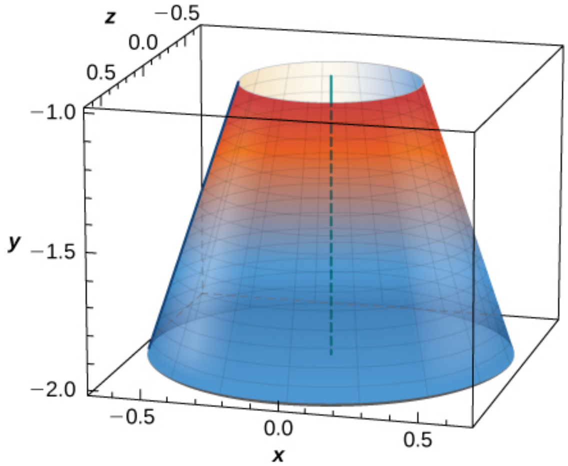
99. $y = x + 2,\ y = 2x - 1,$ and $x = 0$

100. $y = \sqrt[3]{x}$ and $y = x^3$

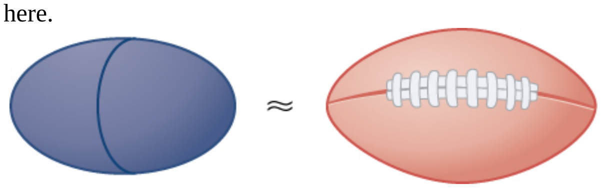
101. $x = e^{2y},\ x = y^2,\ y = 0,$ and $y = \ln(2)$

102. $x = \sqrt{9 - y^2},\ x = e^{-y},\ y = 0,$ and $y = 3$

103. Yogurt containers can be shaped like frustums. Rotate the line $y = \frac{1}{m}x$ around the y-axis to find the volume between $y = a$ and $y = b$.

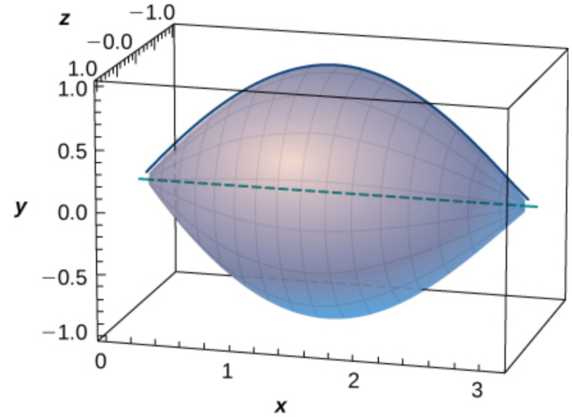


104. Rotate the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ around the x-axis to approximate the volume of a football, as seen here.

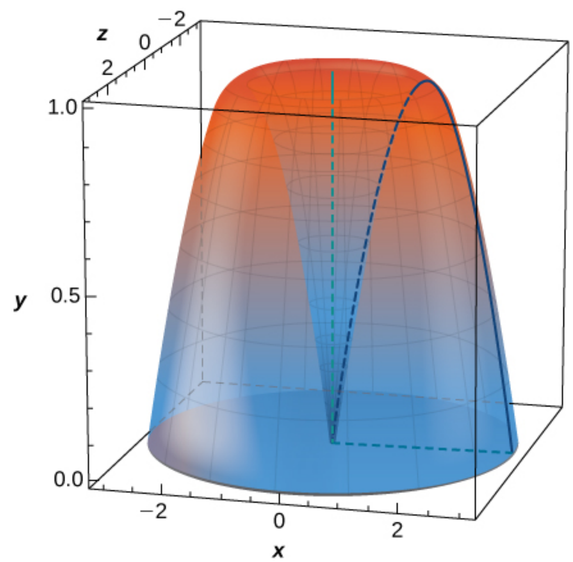


105. Rotate the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ around the y-axis to approximate the volume of a football.

106. A better approximation of the volume of a football is given by the solid that comes from rotating $y = \sin x$ around the x-axis from $x = 0$ to $x = \pi$. What is the volume of this football approximation, as seen here?



107. What is the volume of the Bundt cake that comes from rotating $y = \sin x$ around the y-axis from $x = 0$ to $x = \pi$?

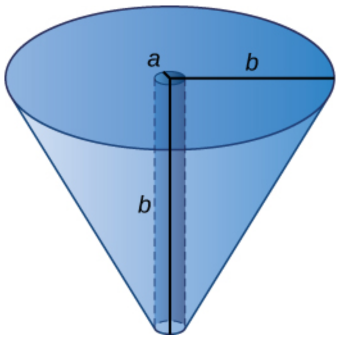


For the following exercises, find the volume of the solid described.

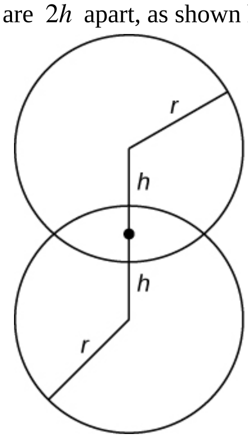
108. The base is the region between $y = x$ and $y = x^2$. Slices perpendicular to the x-axis are semicircles.

109. The base is the region enclosed by the generic ellipse $\left(x^2/a^2\right) + \left(y^2/b^2\right) = 1$. Slices perpendicular to the x-axis are semicircles.

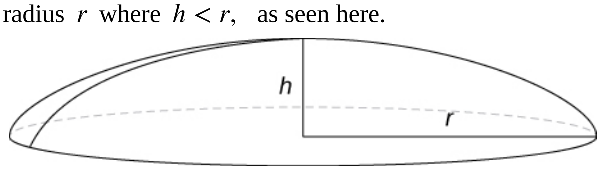
110. Bore a hole of radius a down the axis of a right cone and through the base of radius b, as seen here.



111. Find the volume common to two spheres of radius r with centers that are $2h$ apart, as shown here.



112. Find the volume of a spherical cap of height h and radius r where $h < r$, as seen here.



113. Find the volume of a sphere of radius R with a cap of height h removed from the top, as seen here.