5.7 EXERCISES

In the following exercises, evaluate each integral in terms of an inverse trigonometric function.

391. $\displaystyle\int_0^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}}$

392. $\displaystyle\int_{-1/2}^{1/2} \frac{dx}{1-x^2}$

393. $\displaystyle\int_{\sqrt{3}}^{1} \frac{dx}{1-x^2}$

394. $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{dx}{1+x^2}$

395. $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{|x|\sqrt{x^2-1}}$

396. $\displaystyle\int_1^{2/\sqrt{3}} \frac{dx}{|x|\sqrt{x^2-1}}$

In the following exercises, find each indefinite integral, using appropriate substitutions.

397. $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$

398. $\displaystyle\int \frac{dx}{\sqrt{1-16x^2}}$

399. $\displaystyle\int \frac{dx}{9+x^2}$

400. $\displaystyle\int \frac{dx}{25+16x^2}$

401. $\displaystyle\int \frac{dx}{|x|\sqrt{x^2-9}}$

402. $\displaystyle\int \frac{dx}{|x|\sqrt{4x^2-16}}$

403. Explain the relationship $-\cos^{-1} t + C = \displaystyle\int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t + C$. Is it true, in general, that $\cos^{-1} t = -\sin^{-1} t$?

404. Explain the relationship $\sec^{-1} t + C = \displaystyle\int \frac{dt}{|t|\sqrt{t^2-1}} = -\csc^{-1} t + C$. Is it true, in general, that $\sec^{-1} t = -\csc^{-1} t$?

405. Explain what is wrong with the following integral: $\displaystyle\int_1^2 \frac{dt}{\sqrt{1-t^2}}$.

406. Explain what is wrong with the following integral: $\displaystyle\int_{-1}^{1} \frac{dt}{|t|\sqrt{t^2-1}}$.

In the following exercises, solve for the antiderivative $\int f$ of f with $C = 0$, then use a calculator to graph f and the antiderivative over the given interval $[a, b]$. Identify a value of C such that adding C to the antiderivative recovers the definite integral $F(x) = \displaystyle\int_a^x f(t)dt$.

407. **[T]** $\displaystyle\int \frac{1}{\sqrt{9-x^2}}dx$ over $[-3, 3]$

408. **[T]** $\displaystyle\int \frac{9}{9+x^2}dx$ over $[-6, 6]$

409. **[T]** $\displaystyle\int \frac{\cos x}{4+\sin^2 x}dx$ over $[-6, 6]$

410. **[T]** $\displaystyle\int \frac{e^x}{1+e^{2x}}dx$ over $[-6, 6]$

In the following exercises, compute the antiderivative using appropriate substitutions.

411. $\displaystyle\int \frac{\sin^{-1} t dt}{\sqrt{1-t^2}}$

412. $\displaystyle\int \frac{dt}{\sin^{-1} t\sqrt{1-t^2}}$

413. $\int \frac{\tan^{-1}(2t)}{1+4t^2}dt$

414. $\int \frac{t\tan^{-1}(t^2)}{1+t^4}dt$

415. $\int \frac{\sec^{-1}\left(\frac{t}{2}\right)}{|t|\sqrt{t^2-4}}dt$

416. $\int \frac{t\sec^{-1}(t^2)}{t^2\sqrt{t^4-1}}dt$

In the following exercises, use a calculator to graph the antiderivative $\int f$ with $C = 0$ over the given interval $[a, b]$. Approximate a value of C, if possible, such that adding C to the antiderivative gives the same value as the definite integral $F(x) = \int_a^x f(t)dt$.

417. **[T]** $\int \frac{1}{x\sqrt{x^2-4}}dx$ over $[2, 6]$

418. **[T]** $\int \frac{1}{(2x+2)\sqrt{x}}dx$ over $[0, 6]$

419. **[T]** $\int \frac{(\sin x + x\cos x)}{1+x^2\sin^2 x}dx$ over $[-6, 6]$

420. **[T]** $\int \frac{2e^{-2x}}{\sqrt{1-e^{-4x}}}dx$ over $[0, 2]$

421. **[T]** $\int \frac{1}{x+x\ln^2 x}$ over $[0, 2]$

422. **[T]** $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ over $[-1, 1]$

In the following exercises, compute each integral using appropriate substitutions.

423. $\int \frac{e^t}{\sqrt{1-e^{2t}}}dt$

424. $\int \frac{e^t}{1+e^{2t}}dt$

425. $\int \frac{dt}{t\sqrt{1-\ln^2 t}}$

426. $\int \frac{dt}{t(1+\ln^2 t)}$

427. $\int \frac{\cos^{-1}(2t)}{\sqrt{1-4t^2}}dt$

428. $\int \frac{e^t\cos^{-1}(e^t)}{\sqrt{1-e^{2t}}}dt$

In the following exercises, compute each definite integral.

429. $\int_0^{1/2} \frac{\tan(\sin^{-1} t)}{\sqrt{1-t^2}}dt$

430. $\int_{1/4}^{1/2} \frac{\tan(\cos^{-1} t)}{\sqrt{1-t^2}}dt$

431. $\int_0^{1/2} \frac{\sin(\tan^{-1} t)}{1+t^2}dt$

432. $\int_0^{1/2} \frac{\cos(\tan^{-1} t)}{1+t^2}dt$

433. For $A > 0$, compute $I(A) = \int_{-A}^{A} \frac{dt}{1+t^2}$ and evaluate $\lim_{a \to \infty} I(A)$, the area under the graph of $\frac{1}{1+t^2}$ on $[-\infty, \infty]$.

434. For $1 < B < \infty$, compute $I(B) = \int_1^B \frac{dt}{t\sqrt{t^2-1}}$ and evaluate $\lim_{B \to \infty} I(B)$, the area under the graph of $\frac{1}{t\sqrt{t^2-1}}$ over $[1, \infty)$.

435. Use the substitution $u = \sqrt{2}\cot x$ and the identity $1 + \cot^2 x = \csc^2 x$ to evaluate $\int \frac{dx}{1+\cos^2 x}$. (*Hint:* Multiply the top and bottom of the integrand by $\csc^2 x$.)

436. **[T]** Approximate the points at which the graphs of $f(x) = 2x^2 - 1$ and $g(x) = \left(1 + 4x^2\right)^{-3/2}$ intersect to four decimal places, and approximate the area between their graphs to three decimal places.

437. 47. **[T]** Approximate the points at which the graphs of $f(x) = x^2 - 1$ and $g(x) = \left(x^2 + 1\right)^{\frac{1}{2}}$ intersect to four decimal places, and approximate the area between their graphs to three decimal places.

438. Use the following graph to prove that $\int_0^x \sqrt{1 - t^2}dt = \frac{1}{2}x\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x.$