5.5 EXERCISES

254. Why is u-substitution referred to as *change of variable*?

255. 2. If $f = g \circ h$, when reversing the chain rule, $\frac{d}{dx}(g \circ h)(x) = g'(h(x))h'(x)$, should you take $u = g(x)$ or $u = h(x)$?

In the following exercises, verify each identity using differentiation. Then, using the indicated u-substitution, identify f such that the integral takes the form $\int f(u)du$.

256. $\int x\sqrt{x+1}dx = \frac{2}{15}(x+1)^{3/2}(3x-2) + C;\ u = x+1$

257. For $x > 1$: $\int \frac{x^2}{\sqrt{x-1}}dx = \frac{2}{15}\sqrt{x-1}\left(3x^2 + 4x + 8\right) + C;\ u = x-1$

258. $\int x\sqrt{4x^2+9}dx = \frac{1}{12}\left(4x^2+9\right)^{3/2} + C;\ u = 4x^2+9$

259. $\int \frac{x}{\sqrt{4x^2+9}}dx = \frac{1}{4}\sqrt{4x^2+9} + C;\ u = 4x^2+9$

260. $\int \frac{x}{(4x^2+9)^2}dx = -\frac{1}{8(4x^2+9)};\ u = 4x^2+9$

In the following exercises, find the antiderivative using the indicated substitution.

261. $\int (x+1)^4 dx;\ u = x+1$

262. $\int (x-1)^5 dx;\ u = x-1$

263. $\int (2x-3)^{-7} dx;\ u = 2x-3$

264. $\int (3x-2)^{-11} dx;\ u = 3x-2$

265. $\int \frac{x}{\sqrt{x^2+1}}dx;\ u = x^2+1$

266. $\int \frac{x}{\sqrt{1-x^2}}dx;\ u = 1-x^2$

267. $\int (x-1)\left(x^2-2x\right)^3 dx;\ u = x^2-2x$

268. $\int \left(x^2-2x\right)\left(x^3-3x^2\right)^2 dx;\ u = x^3-3x^2$

269. $\int \cos^3\theta d\theta;\ u = \sin\theta$ $(Hint\colon \cos^2\theta = 1 - \sin^2\theta)$

270. $\int \sin^3\theta d\theta;\ u = \cos\theta$ $(Hint\colon \sin^2\theta = 1 - \cos^2\theta)$

In the following exercises, use a suitable change of variables to determine the indefinite integral.

271. $\int x(1-x)^{99} dx$

272. $\int t\left(1-t^2\right)^{10} dt$

273. $\int (11x-7)^{-3} dx$

274. $\int (7x-11)^4 dx$

275. $\int \cos^3\theta \sin\theta d\theta$

276. $\int \sin^7\theta \cos\theta d\theta$

277. $\int \cos^2(\pi t)\sin(\pi t)dt$

278. $\int \sin^2 x \cos^3 x dx$ $(Hint\colon \sin^2 x + \cos^2 x = 1)$

279. $\int t\sin\left(t^2\right)\cos\left(t^2\right)dt$

280. $\int t^2\cos^2\left(t^3\right)\sin\left(t^3\right)dt$

281. $\int \frac{x^2}{\left(x^3-3\right)^2}dx$

282. $\int \frac{x^3}{\sqrt{1-x^2}}dx$

283. $\int \frac{y^5}{\left(1-y^3\right)^{3/2}}dy$

284. $\int \cos\theta(1-\cos\theta)^{99}\sin\theta d\theta$

285. $\int \left(1-\cos^3\theta\right)^{10}\cos^2\theta\sin\theta d\theta$

286. $\int (\cos\theta-1)\left(\cos^2\theta-2\cos\theta\right)^3\sin\theta d\theta$

287. $\int \left(\sin^2\theta-2\sin\theta\right)\left(\sin^3\theta-3\sin^2\theta\right)^3\cos\theta d\theta$

In the following exercises, use a calculator to estimate the area under the curve using left Riemann sums with 50 terms, then use substitution to solve for the exact answer.

288. **[T]** $y=3(1-x)^2$ over $[0, 2]$

289. **[T]** $y=x\left(1-x^2\right)^3$ over $[-1, 2]$

290. **[T]** $y=\sin x(1-\cos x)^2$ over $[0, \pi]$

291. **[T]** $y=\frac{x}{\left(x^2+1\right)^2}$ over $[-1, 1]$

In the following exercises, use a change of variables to evaluate the definite integral.

292. $\int_0^1 x\sqrt{1-x^2}dx$

293. $\int_0^1 \frac{x}{\sqrt{1+x^2}}dx$

294. $\int_0^2 \frac{t^2}{\sqrt{5+t^2}}dt$

295. $\int_0^1 \frac{t^2}{\sqrt{1+t^3}}dt$

296. $\int_0^{\pi/4} \sec^2\theta\tan\theta d\theta$

297. $\int_0^{\pi/4} \frac{\sin\theta}{\cos^4\theta}d\theta$

In the following exercises, evaluate the indefinite integral $\int f(x)dx$ with constant $C=0$ using u-substitution. Then, graph the function and the antiderivative over the indicated interval. If possible, estimate a value of C that would need to be added to the antiderivative to make it equal to the definite integral $F(x)=\int_a^x f(t)dt,$ with a the left endpoint of the given interval.

298. **[T]** $\int (2x+1)e^{x^2+x-6}dx$ over $[-3, 2]$

299. **[T]** $\int \frac{\cos(\ln(2x))}{x}dx$ on $[0, 2]$

300. **[T]** $\int \frac{3x^2+2x+1}{\sqrt{x^3+x^2+x+4}}dx$ over $[-1, 2]$

301. **[T]** $\int \frac{\sin x}{\cos^3 x}dx$ over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

302. **[T]** $\int (x+2)e^{-x^2-4x+3}dx$ over $[-5, 1]$

303. **[T]** $\int 3x^2\sqrt{2x^3+1}dx$ over $[0, 1]$

304. If $h(a)=h(b)$ in $\int_a^b g'(h(x))h(x)dx,$ what can you say about the value of the integral?

305. Is the substitution $u=1-x^2$ in the definite integral $\int_0^2 \frac{x}{1-x^2}dx$ okay? If not, why not?

In the following exercises, use a change of variables to show that each definite integral is equal to zero.

306. $\int_0^{\pi} \cos^2(2\theta)\sin(2\theta)d\theta$

307. $\int_0^{\sqrt{\pi}} t\cos\left(t^2\right)\sin\left(t^2\right)dt$

308. $\int_0^1 (1-2t)dt$

309. $\displaystyle\int_0^1 \frac{1-2t}{\left(1+\left(t-\frac{1}{2}\right)^2\right)}dt$

310. $\displaystyle\int_0^{\pi} \sin\left(\left(t-\frac{\pi}{2}\right)^3\right)\cos\left(t-\frac{\pi}{2}\right)dt$

311. $\displaystyle\int_0^2 (1-t)\cos(\pi t)dt$

312. $\displaystyle\int_{\pi/4}^{3\pi/4} \sin^2 t\cos t dt$

313. Show that the average value of $f(x)$ over an interval $[a, b]$ is the same as the average value of $f(cx)$ over the interval $\left[\frac{a}{c}, \frac{b}{c}\right]$ for $c > 0$.

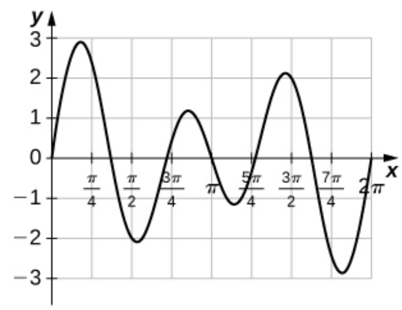
314. Find the area under the graph of $f(t) = \dfrac{t}{\left(1+t^2\right)^a}$ between $t = 0$ and $t = x$ where $a > 0$ and $a \neq 1$ is fixed, and evaluate the limit as $x \to \infty$.

315. Find the area under the graph of $g(t) = \dfrac{t}{\left(1-t^2\right)^a}$ between $t = 0$ and $t = x$, where $0 < x < 1$ and $a > 0$ is fixed. Evaluate the limit as $x \to 1$.

316. The area of a semicircle of radius 1 can be expressed as $\displaystyle\int_{-1}^{1}\sqrt{1-x^2}dx$. Use the substitution $x = \cos t$ to express the area of a semicircle as the integral of a trigonometric function. You do not need to compute the integral.

317. The area of the top half of an ellipse with a major axis that is the *x*-axis from $x = a$ to *a* and with a minor axis that is the *y*-axis from $y = -b$ to *b* can be written as $\displaystyle\int_{-a}^{a} b\sqrt{1-\frac{x^2}{a^2}}dx$. Use the substitution $x = a\cos t$ to express this area in terms of an integral of a trigonometric function. You do not need to compute the integral.

318. **[T]** The following graph is of a function of the form $f(t) = a\sin(nt) + b\sin(mt)$. Estimate the coefficients *a* and *b*, and the frequency parameters *n* and *m*. Use these estimates to approximate $\displaystyle\int_0^{\pi} f(t)dt$.



319. **[T]** The following graph is of a function of the form $f(x) = a\cos(nt) + b\cos(mt)$. Estimate the coefficients *a* and *b* and the frequency parameters *n* and *m*. Use these estimates to approximate $\displaystyle\int_0^{\pi} f(t)dt$.