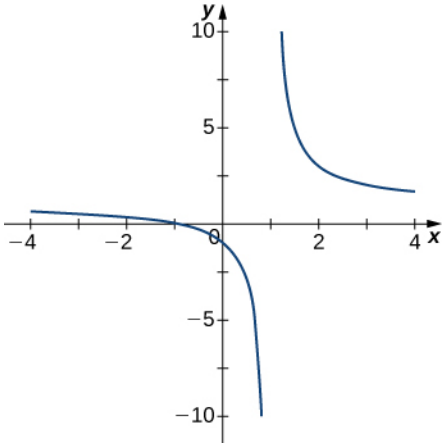
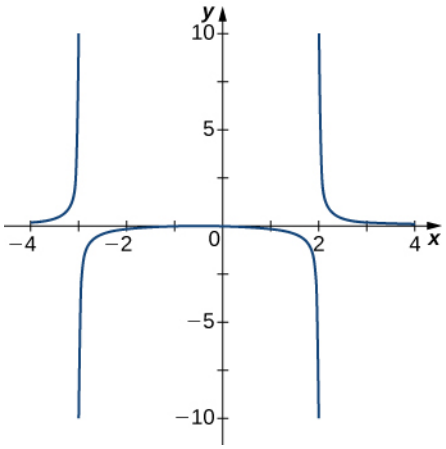
4.6 EXERCISES

For the following exercises, examine the graphs. Identify where the vertical asymptotes are located.

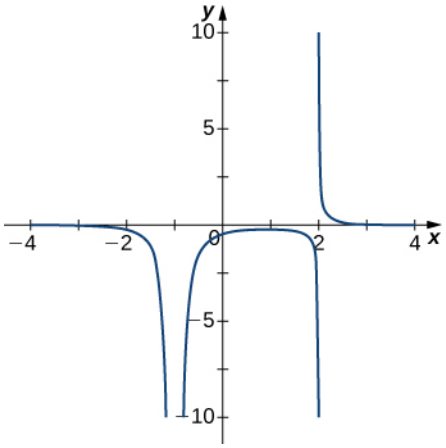
251.



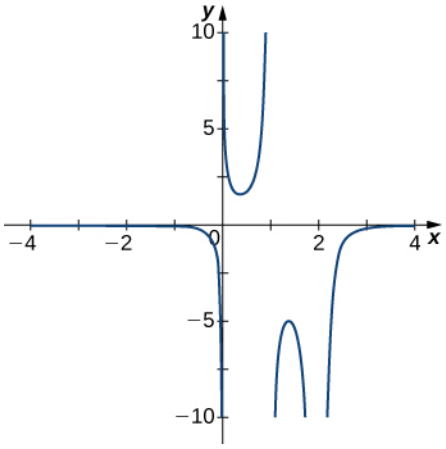
252.



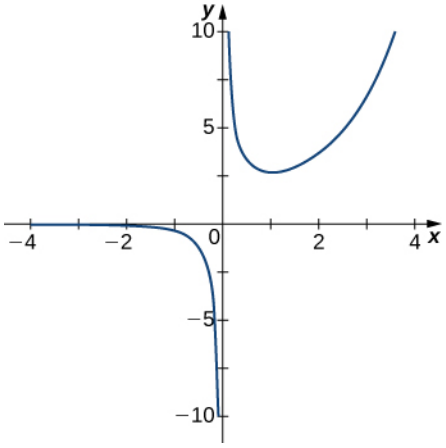
253.



254.



255.



For the following functions $f(x)$, determine whether there is an asymptote at $x = a$. Justify your answer without graphing on a calculator.

256. $f(x) = \dfrac{x+1}{x^2+5x+4}$, $a = -1$

257. $f(x) = \dfrac{x}{x-2}$, $a = 2$

258. $f(x) = (x+2)^{3/2}$, $a = -2$

259. $f(x) = (x-1)^{-1/3}$, $a = 1$

260. $f(x) = 1 + x^{-2/5}$, $a = 1$

For the following exercises, evaluate the limit.

261. $\lim\limits_{x \to \infty} \dfrac{1}{3x+6}$

262. $\lim_{x \to \infty} \frac{2x-5}{4x}$

263. $\lim_{x \to \infty} \frac{x^2-2x+5}{x+2}$

264. $\lim_{x \to -\infty} \frac{3x^3-2x}{x^2+2x+8}$

265. $\lim_{x \to -\infty} \frac{x^4-4x^3+1}{2-2x^2-7x^4}$

266. $\lim_{x \to \infty} \frac{3x}{\sqrt{x^2+1}}$

267. $\lim_{x \to -\infty} \frac{\sqrt{4x^2-1}}{x+2}$

268. $\lim_{x \to \infty} \frac{4x}{\sqrt{x^2-1}}$

269. $\lim_{x \to -\infty} \frac{4x}{\sqrt{x^2-1}}$

270. $\lim_{x \to \infty} \frac{2\sqrt{x}}{x-\sqrt{x}+1}$

For the following exercises, find the horizontal and vertical asymptotes.

271. $f(x) = x - \frac{9}{x}$

272. $f(x) = \frac{1}{1-x^2}$

273. $f(x) = \frac{x^3}{4-x^2}$

274. $f(x) = \frac{x^2+3}{x^2+1}$

275. $f(x) = \sin(x)\sin(2x)$

276. $f(x) = \cos x + \cos(3x) + \cos(5x)$

277. $f(x) = \frac{x\sin(x)}{x^2-1}$

278. $f(x) = \frac{x}{\sin(x)}$

279. $f(x) = \frac{1}{x^3+x^2}$

280. $f(x) = \frac{1}{x-1} - 2x$

281. $f(x) = \frac{x^3+1}{x^3-1}$

282. $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$

283. $f(x) = x - \sin x$

284. $f(x) = \frac{1}{x} - \sqrt{x}$

For the following exercises, construct a function $f(x)$ that has the given asymptotes.

285. $x = 1$ and $y = 2$

286. $x = 1$ and $y = 0$

287. $y = 4, \quad x = -1$

288. $x = 0$

For the following exercises, graph the function on a graphing calculator on the window $x = [-5, 5]$ and estimate the horizontal asymptote or limit. Then, calculate the actual horizontal asymptote or limit.

289. **[T]** $f(x) = \frac{1}{x+10}$

290. **[T]** $f(x) = \frac{x+1}{x^2+7x+6}$

291. **[T]** $\lim_{x \to -\infty} x^2 + 10x + 25$

292. **[T]** $\lim_{x \to -\infty} \frac{x+2}{x^2+7x+6}$

293. **[T]** $\lim_{x \to \infty} \frac{3x+2}{x+5}$

For the following exercises, draw a graph of the functions without using a calculator. Be sure to notice all important features of the graph: local maxima and minima, inflection points, and asymptotic behavior.

294. $y = 3x^2 + 2x + 4$

295. $y = x^3 - 3x^2 + 4$

296. $y = \frac{2x+1}{x^2+6x+5}$

297. $y = \frac{x^3+4x^2+3x}{3x+9}$

298. $y = \dfrac{x^2 + x - 2}{x^2 - 3x - 4}$

299. $y = \sqrt[4]{x^2 - 5x + 4}$

300. $y = 2x\sqrt[4]{16 - x^2}$

301. $y = \dfrac{\cos x}{x}$, on $x = [-2\pi, 2\pi]$

302. $y = e^x - x^3$

303. $y = x\tan x$, $x = [-\pi, \pi]$

304. $y = x\ln(x)$, $x > 0$

305. $y = x^2\sin(x)$, $x = [-2\pi, 2\pi]$

306. For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $y = 2$ then the polynomials $P(x)$ and $Q(x)$ must have what relation?

307. For $f(x) = \dfrac{P(x)}{Q(x)}$ to have an asymptote at $x = 0$, then the polynomials $P(x)$ and $Q(x)$. must have what relation?

308. If $f'(x)$ has asymptotes at $y = 3$ and $x = 1$, then $f(x)$ has what asymptotes?

309. Both $f(x) = \dfrac{1}{(x-1)}$ and $g(x) = \dfrac{1}{(x-1)^2}$ have asymptotes at $x = 1$ and $y = 0$. What is the most obvious difference between these two functions?

310. True or false: Every ratio of polynomials has vertical asymptotes.