4.5 EXERCISES

194. If c is a critical point of $f(x)$, when is there no local maximum or minimum at c? Explain.

195. For the function $y = x^3$, is $x = 0$ both an inflection point and a local maximum/minimum?

196. For the function $y = x^3$, is $x = 0$ an inflection point?

197. Is it possible for a point c to be both an inflection point and a local extrema of a twice differentiable function?

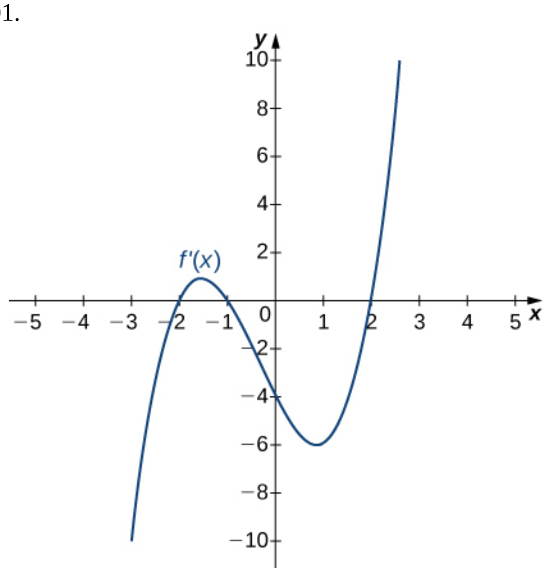
198. Why do you need continuity for the first derivative test? Come up with an example.

199. Explain whether a concave-down function has to cross $y = 0$ for some value of x.

200. Explain whether a polynomial of degree 2 can have an inflection point.

For the following exercises, analyze the graphs of f', then list all intervals where f is increasing or decreasing.

201.



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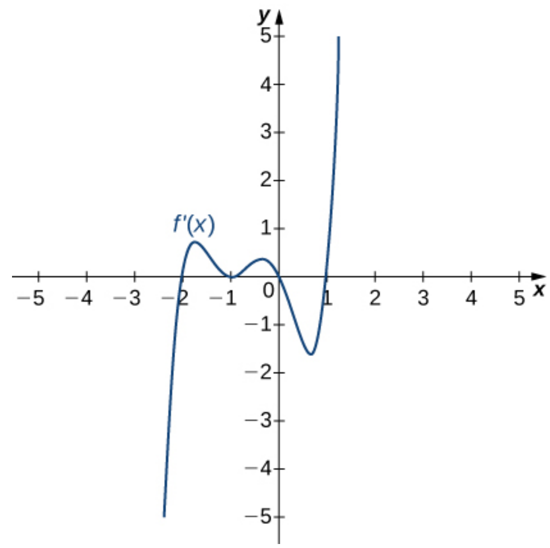

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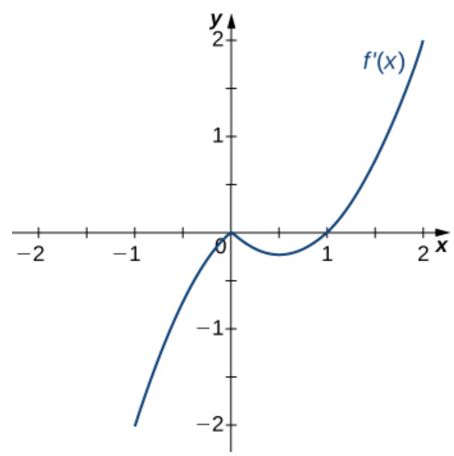

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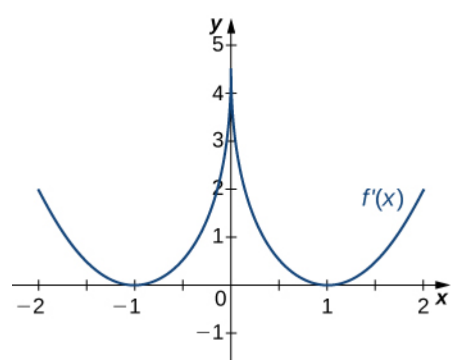

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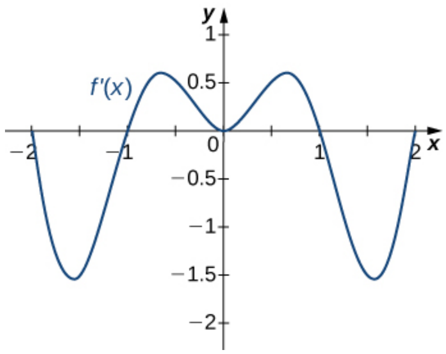
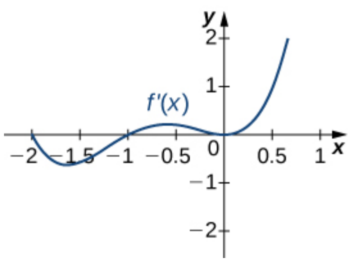


For the following exercises, analyze the graphs of f', then list all intervals where

a. f is increasing and decreasing and

b. the minima and maxima are located.

206.



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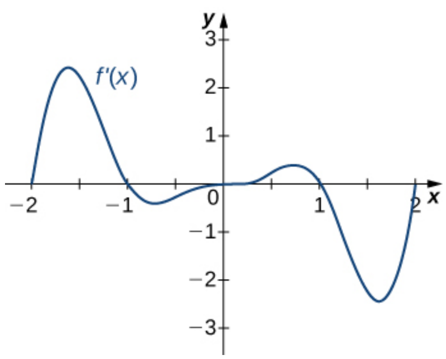

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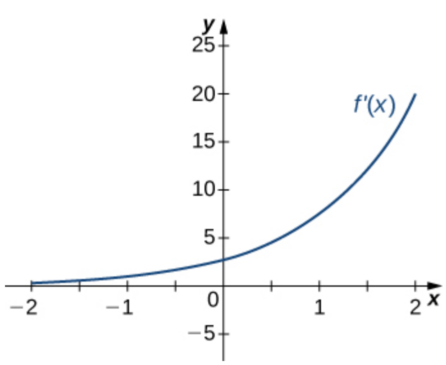

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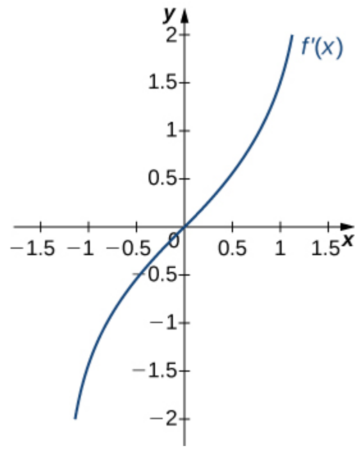

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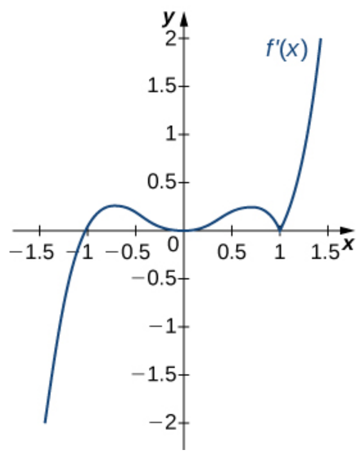
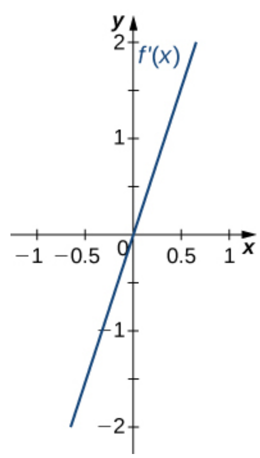
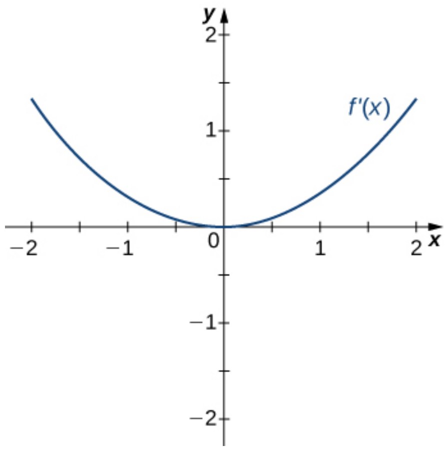


For the following exercises, analyze the graphs of f', then list all inflection points and intervals f that are concave up and concave down.

211.



212.



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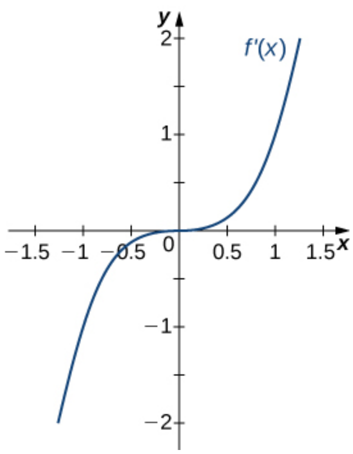

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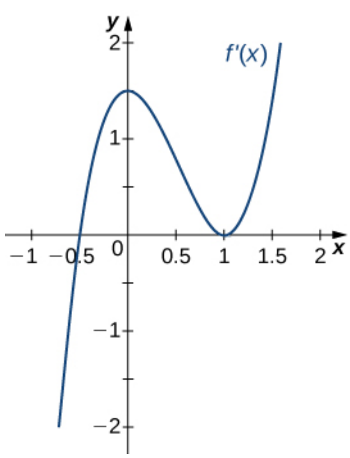
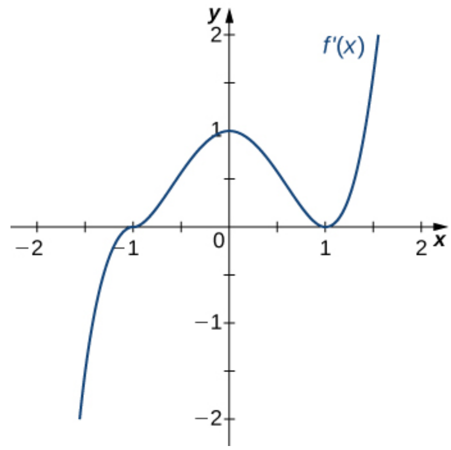


215.



For the following exercises, draw a graph that satisfies the given specifications for the domain $x = [-3, 3]$. The function does not have to be continuous or differentiable.

216. $f(x) > 0,\ f'(x) > 0$ over $x > 1,\ -3 < x < 0,\ f'(x) = 0$ over $0 < x < 1$

217. $f'(x) > 0$ over $x > 2,\ -3 < x < -1,\ f'(x) < 0$ over $-1 < x < 2,\ f''(x) < 0$ for all x

218. $f''(x) < 0$ over $-1 < x < 1,\ f''(x) > 0,\ -3 < x < -1,\ 1 < x < 3,$ local maximum at $x = 0,$ local minima at $x = \pm 2$

219. There is a local maximum at $x = 2,$ local minimum at $x = 1,$ and the graph is neither concave up nor concave down.

220. There are local maxima at $x = \pm 1,$ the function is concave up for all $x,$ and the function remains positive for all $x.$

For the following exercises, determine

a. intervals where f is increasing or decreasing and

b. local minima and maxima of f.

221. $f(x) = \sin x + \sin^3 x$ over $-\pi < x < \pi$

222. $f(x) = x^2 + \cos x$

For the following exercises, determine a. intervals where f is concave up or concave down, and b. the inflection points of f.

223. $f(x) = x^3 - 4x^2 + x + 2$

For the following exercises, determine

a. intervals where f is increasing or decreasing,
b. local minima and maxima of f,
c. intervals where f is concave up and concave down, and
d. the inflection points of f.

224. $f(x) = x^2 - 6x$

225. $f(x) = x^3 - 6x^2$

226. $f(x) = x^4 - 6x^3$

227. $f(x) = x^{11} - 6x^{10}$

228. $f(x) = x + x^2 - x^3$

229. $f(x) = x^2 + x + 1$

230. $f(x) = x^3 + x^4$

For the following exercises, determine

a. intervals where f is increasing or decreasing,
b. local minima and maxima of f,
c. intervals where f is concave up and concave down, and
d. the inflection points of f. Sketch the curve, then use a calculator to compare your answer. If you cannot determine the exact answer analytically, use a calculator.

231. **[T]** $f(x) = \sin(\pi x) - \cos(\pi x)$ over $x = [-1, 1]$

232. **[T]** $f(x) = x + \sin(2x)$ over $x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

233. **[T]** $f(x) = \sin x + \tan x$ over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

234. **[T]** $f(x) = (x-2)^2(x-4)^2$

235. **[T]** $f(x) = \frac{1}{1-x}, x \neq 1$

236. **[T]** $f(x) = \frac{\sin x}{x}$ over $x = [2\pi, 0) \cup (0, 2\pi]$

237. $f(x) = \sin(x)e^x$ over $x = [-\pi, \pi]$

238. $f(x) = \ln x\sqrt{x}, x > 0$

239. $f(x) = \frac{1}{4}\sqrt{x} + \frac{1}{x}, x > 0$

240. $f(x) = \frac{e^x}{x}, x \neq 0$

For the following exercises, interpret the sentences in terms of f, f', and f''.

241. The population is growing more slowly. Here f is the population.

242. A bike accelerates faster, but a car goes faster. Here $f =$ Bike's position minus Car's position.

243. The airplane lands smoothly. Here f is the plane's altitude.

244. Stock prices are at their peak. Here f is the stock price.

245. The economy is picking up speed. Here f is a measure of the economy, such as GDP.

For the following exercises, consider a third-degree polynomial $f(x)$, which has the properties $f'(1) = 0, f'(3) = 0$. Determine whether the following statements are *true or false*. Justify your answer.

246. $f(x) = 0$ for some $1 \le x \le 3$

247. $f''(x) = 0$ for some $1 \le x \le 3$

248. There is no absolute maximum at $x = 3$

249. If $f(x)$ has three roots, then it has 1 inflection point.

250. If $f(x)$ has one inflection point, then it has three real roots.