3.6 EXERCISES

For the following exercises, given $y = f(u)$ and $u = g(x)$, find $\frac{dy}{dx}$ by using Leibniz's notation for the chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

214. $y = 3u - 6,\ u = 2x^2$

215. $y = 6u^3,\ u = 7x - 4$

216. $y = \sin u,\ u = 5x - 1$

217. $y = \cos u,\ u = \frac{-x}{8}$

218. $y = \tan u,\ u = 9x + 2$

219. $y = \sqrt{4u + 3},\ u = x^2 - 6x$

For each of the following exercises,

a. decompose each function in the form $y = f(u)$ and $u = g(x)$, and

b. find $\frac{dy}{dx}$ as a function of x.

220. $y = (3x - 2)^6$

221. $y = \left(3x^2 + 1\right)^3$

222. $y = \sin^5(x)$

223. $y = \left(\frac{x}{7} + \frac{7}{x}\right)^7$

224. $y = \tan(\sec x)$

225. $y = \csc(\pi x + 1)$

226. $y = \cot^2 x$

227. $y = -6\sin^{-3} x$

For the following exercises, find $\frac{dy}{dx}$ for each function.

228. $y = \left(3x^2 + 3x - 1\right)^4$

229. $y = (5 - 2x)^{-2}$

230. $y = \cos^3(\pi x)$

231. $y = \left(2x^3 - x^2 + 6x + 1\right)^3$

232. $y = \frac{1}{\sin^2(x)}$

233. $y = (\tan x + \sin x)^{-3}$

234. $y = x^2 \cos^4 x$

235. $y = \sin(\cos 7x)$

236. $y = \sqrt{6 + \sec \pi x^2}$

237. $y = \cot^3(4x + 1)$

238. Let $y = [f(x)]^3$ and suppose that $f'(1) = 4$ and $\frac{dy}{dx} = 10$ for $x = 1$. Find $f(1)$.

239. Let $y = \left(f(x) + 5x^2\right)^4$ and suppose that $f(-1) = -4$ and $\frac{dy}{dx} = 3$ when $x = -1$. Find $f'(-1)$

240. Let $y = (f(u) + 3x)^2$ and $u = x^3 - 2x$. If $f(4) = 6$ and $\frac{dy}{dx} = 18$ when $x = 2$, find $f'(4)$.

241. **[T]** Find the equation of the tangent line to $y = -\sin\left(\frac{x}{2}\right)$ at the origin. Use a calculator to graph the function and the tangent line together.

242. **[T]** Find the equation of the tangent line to $y = \left(3x + \frac{1}{x}\right)^2$ at the point $(1, 16)$. Use a calculator to graph the function and the tangent line together.

243. Find the x-coordinates at which the tangent line to $y = \left(x - \frac{6}{x}\right)^8$ is horizontal.

244. **[T]** Find an equation of the line that is normal to $g(\theta) = \sin^2(\pi\theta)$ at the point $\left(\frac{1}{4}, \frac{1}{2}\right)$. Use a calculator to graph the function and the normal line together.

For the following exercises, use the information in the following table to find $h'(a)$ at the given value for a.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	5	0	2
1	1	−2	3	0
2	4	4	1	−1
3	3	−3	2	3

245. $h(x) = f(g(x)); a = 0$

246. $h(x) = g(f(x)); a = 0$

247. $h(x) = \left(x^4 + g(x)\right)^{-2}; a = 1$

248. $h(x) = \left(\frac{f(x)}{g(x)}\right)^2; a = 3$

249. $h(x) = f(x + f(x)); a = 1$

250. $h(x) = (1 + g(x))^3; a = 2$

251. $h(x) = g\left(2 + f\left(x^2\right)\right); a = 1$

252. $h(x) = f(g(\sin x)); a = 0$

253. **[T]** The position function of a freight train is given by $s(t) = 100(t + 1)^{-2}$, with s in meters and t in seconds. At time $t = 6$ s, find the train's

a. velocity and
b. acceleration.
c. Using a. and b. is the train speeding up or slowing down?

254. **[T]** A mass hanging from a vertical spring is in simple harmonic motion as given by the following position function, where t is measured in seconds and s is in inches: $s(t) = -3\cos\left(\pi t + \frac{\pi}{4}\right)$.

a. Determine the position of the spring at $t = 1.5$ s.
b. Find the velocity of the spring at $t = 1.5$ s.

255. **[T]** The total cost to produce x boxes of Thin Mint Girl Scout cookies is C dollars, where $C = 0.0001x^3 - 0.02x^2 + 3x + 300.$ In t weeks production is estimated to be $x = 1600 + 100t$ boxes.

a. Find the marginal cost $C'(x)$.
b. Use Leibniz's notation for the chain rule, $\frac{dC}{dt} = \frac{dC}{dx} \cdot \frac{dx}{dt}$, to find the rate with respect to time t that the cost is changing.
c. Use b. to determine how fast costs are increasing when $t = 2$ weeks. Include units with the answer.

256. **[T]** The formula for the area of a circle is $A = \pi r^2$, where r is the radius of the circle. Suppose a circle is expanding, meaning that both the area A and the radius r (in inches) are expanding.

a. Suppose $r = 2 - \frac{100}{(t + 7)^2}$ where t is time in seconds. Use the chain rule $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ to find the rate at which the area is expanding.
b. Use a. to find the rate at which the area is expanding at $t = 4$ s.

257. **[T]** The formula for the volume of a sphere is $S = \frac{4}{3}\pi r^3$, where r (in feet) is the radius of the sphere. Suppose a spherical snowball is melting in the sun.

a. Suppose $r = \frac{1}{(t + 1)^2} - \frac{1}{12}$ where t is time in minutes. Use the chain rule $\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$ to find the rate at which the snowball is melting.
b. Use a. to find the rate at which the volume is changing at $t = 1$ min.

258. **[T]** The daily temperature in degrees Fahrenheit of Phoenix in the summer can be modeled by the function $T(x) = 94 - 10\cos\left[\frac{\pi}{12}(x - 2)\right]$, where x is hours after midnight. Find the rate at which the temperature is changing at 4 p.m.

259. **[T]** The depth (in feet) of water at a dock changes with the rise and fall of tides. The depth is modeled by the function $D(t) = 5\sin\left(\frac{\pi}{6}t - \frac{7\pi}{6}\right) + 8$, where t is the number of hours after midnight. Find the rate at which the depth is changing at 6 a.m.