3.9 EXERCISES

For the following exercises, find $f'(x)$ for each function.

331. $f(x) = x^2 e^x$

332. $f(x) = \frac{e^{-x}}{x}$

333. $f(x) = e^{x^3 \ln x}$

334. $f(x) = \sqrt{e^{2x} + 2x}$

335. $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

336. $f(x) = \frac{10^x}{\ln 10}$

337. $f(x) = 2^{4x} + 4x^2$

338. $f(x) = 3^{\sin 3\text{x}}$

339. $f(x) = x^{\pi} \cdot \pi^x$

340. $f(x) = \ln\left(4x^3 + x\right)$

341. $f(x) = \ln\sqrt{5x - 7}$

342. $f(x) = x^2 \ln 9x$

343. $f(x) = \log(\sec x)$

344. $f(x) = \log_7\left(6x^4 + 3\right)^5$

345. $f(x) = 2^x \cdot \log_3 7^{x^2 - 4}$

For the following exercises, use logarithmic differentiation to find $\frac{dy}{dx}$.

346. $y = x^{\sqrt{x}}$

347. $y = (\sin 2x)^{4x}$

348. $y = (\ln x)^{\ln x}$

349. $y = x^{\log_2 x}$

350. $y = \left(x^2 - 1\right)^{\ln x}$

351. $y = x^{\cot x}$

352. $y = \frac{x + 11}{\sqrt[3]{x^2 - 4}}$

353. $y = x^{-1/2}\left(x^2 + 3\right)^{2/3}(3x - 4)^4$

354. **[T]** Find an equation of the tangent line to the graph of $f(x) = 4xe^{\left(x^2 - 1\right)}$ at the point where $x = -1$. Graph both the function and the tangent line.

355. **[T]** Find the equation of the line that is normal to the graph of $f(x) = x \cdot 5^x$ at the point where $x = 1$. Graph both the function and the normal line.

356. **[T]** Find the equation of the tangent line to the graph of $x^3 - x\ln y + y^3 = 2x + 5$ at the point where $x = 2$. (*Hint*: Use implicit differentiation to find $\frac{dy}{dx}$.) Graph both the curve and the tangent line.

357. Consider the function $y = x^{1/x}$ for $x > 0$.
    a. Determine the points on the graph where the tangent line is horizontal.
    b. Determine the points on the graph where $y' > 0$ and those where $y' < 0$.

358. The formula $I(t) = \frac{\sin t}{e^t}$ is the formula for a decaying alternating current.

a. Complete the following table with the appropriate values.

t	$\frac{\sin t}{e^t}$
0	(i)
$\frac{\pi}{2}$	(ii)
π	(iii)
$\frac{3\pi}{2}$	(iv)
2π	(v)
$\frac{5\pi}{2}$	(vi)
3π	(vii)
$\frac{7\pi}{2}$	(viii)
4π	(ix)

b. Using only the values in the table, determine where the tangent line to the graph of $I(t)$ is horizontal.

359. **[T]** The population of Toledo, Ohio, in 2000 was approximately 500,000. Assume the population is increasing at a rate of 5% per year.

a. Write the exponential function that relates the total population as a function of t.
b. Use a. to determine the rate at which the population is increasing in t years.
c. Use b. to determine the rate at which the population is increasing in 10 years.

360. **[T]** An isotope of the element erbium has a half-life of approximately 12 hours. Initially there are 9 grams of the isotope present.

a. Write the exponential function that relates the amount of substance remaining as a function of t, measured in hours.
b. Use a. to determine the rate at which the substance is decaying in t hours.
c. Use b. to determine the rate of decay at $t = 4$ hours.

361. **[T]** The number of cases of influenza in New York City from the beginning of 1960 to the beginning of 1961 is modeled by the function $N(t) = 5.3e^{0.093t^2 - 0.87t}, (0 \le t \le 4)$, where $N(t)$ gives the number of cases (in thousands) and t is measured in years, with $t = 0$ corresponding to the beginning of 1960.

a. Show work that evaluates $N(0)$ and $N(4)$. Briefly describe what these values indicate about the disease in New York City.
b. Show work that evaluates $N'(0)$ and $N'(3)$. Briefly describe what these values indicate about the disease in New York City.

362. **[T]** The *relative rate of change* of a differentiable function $y = f(x)$ is given by $\frac{100 \cdot f'(x)}{f(x)}\%$. One model for population growth is a Gompertz growth function, given by $P(x) = ae^{-b \cdot e^{-cx}}$ where $a, b,$ and c are constants.

a. Find the relative rate of change formula for the generic Gompertz function.
b. Use a. to find the relative rate of change of a population in $x = 20$ months when $a = 204,\ b = 0.0198,$ and $c = 0.15$.
c. Briefly interpret what the result of b. means.

For the following exercises, use the population of New York City from 1790 to 1860, given in the following table.

Years since 1790	Population
0	33,131
10	60,515
20	96,373
30	123,706
40	202,300
50	312,710
60	515,547
70	813,669

Table 3.8 New York City Population Over Time **Source: http://en.wikipedia.org/wiki/Largest_cities_in_the_United_States_by_population_by_decade.**

363. **[T]** Using a computer program or a calculator, fit a growth curve to the data of the form $p = ab^t$.

364. **[T]** Using the exponential best fit for the data, write a table containing the derivatives evaluated at each year.

365. **[T]** Using the exponential best fit for the data, write a table containing the second derivatives evaluated at each year.

366. **[T]** Using the tables of first and second derivatives and the best fit, answer the following questions:

a. Will the model be accurate in predicting the future population of New York City? Why or why not?

b. Estimate the population in 2010. Was the prediction correct from a.?