3.2 EXERCISES

For the following exercises, use the definition of a derivative to find $f'(x)$.

54. $f(x) = 6$

55. $f(x) = 2 - 3x$

56. $f(x) = \frac{2x}{7} + 1$

57. $f(x) = 4x^2$

58. $f(x) = 5x - x^2$

59. $f(x) = \sqrt{2x}$

60. $f(x) = \sqrt{x - 6}$

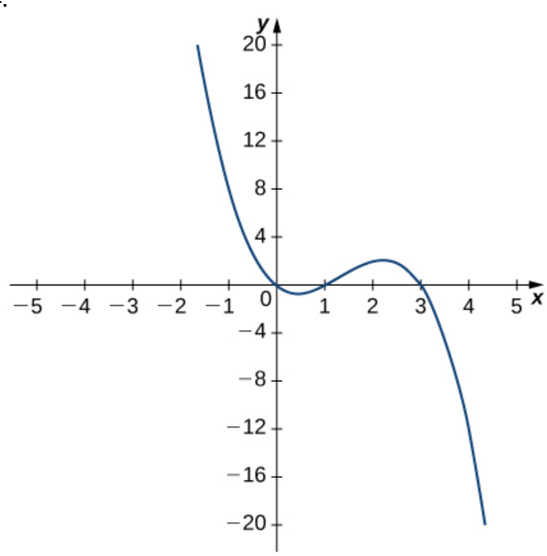
61. $f(x) = \frac{9}{x}$

62. $f(x) = x + \frac{1}{x}$

63. $f(x) = \frac{1}{\sqrt{x}}$

For the following exercises, use the graph of $y = f(x)$ to sketch the graph of its derivative $f'(x)$.

64.



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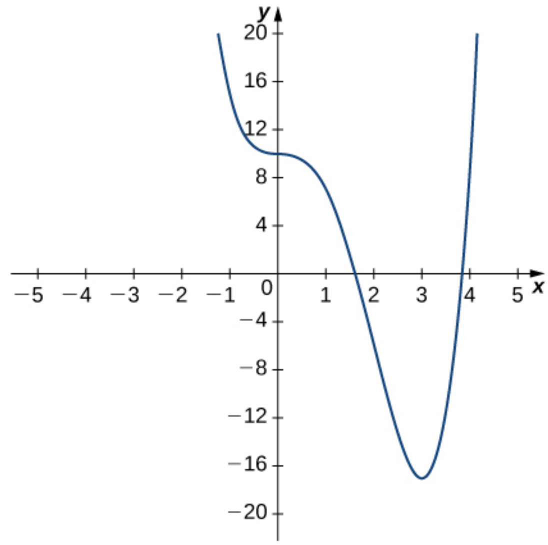

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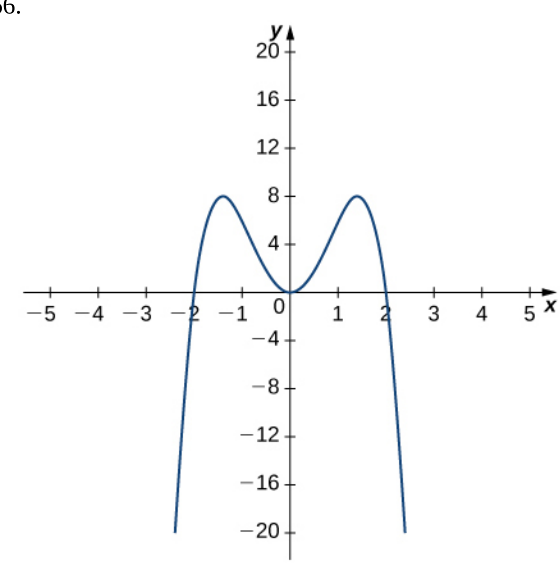

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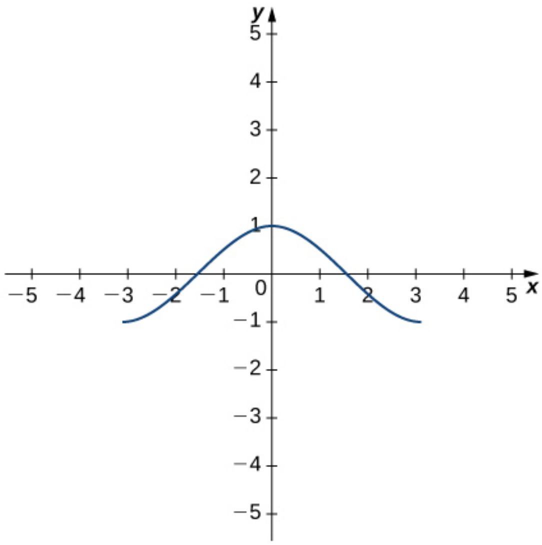


For the following exercises, the given limit represents the derivative of a function $y = f(x)$ at $x = a$. Find $f(x)$ and a.

68. $\lim_{h \to 0} \frac{(1 + h)^{2/3} - 1}{h}$

69. $\lim_{h \to 0} \frac{\left[3(2 + h)^2 + 2\right] - 14}{h}$

70. $\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}$

71. $\lim_{h \to 0} \frac{(2 + h)^4 - 16}{h}$

72. $\lim_{h \to 0} \frac{[2(3 + h)^2 - (3 + h)] - 15}{h}$

73. $\lim_{h \to 0} \frac{e^h - 1}{h}$

For the following functions,

a. sketch the graph and

b. use the definition of a derivative to show that the function is not differentiable at $x = 1$.

74. $f(x) = \begin{cases} 2\sqrt{x}, & 0 \le x \le 1 \\ 3x - 1, & x > 1 \end{cases}$

75. $f(x) = \begin{cases} 3, & x < 1 \\ 3x, & x \ge 1 \end{cases}$

76. $f(x) = \begin{cases} -x^2 + 2, & x \le 1 \\ x, & x > 1 \end{cases}$

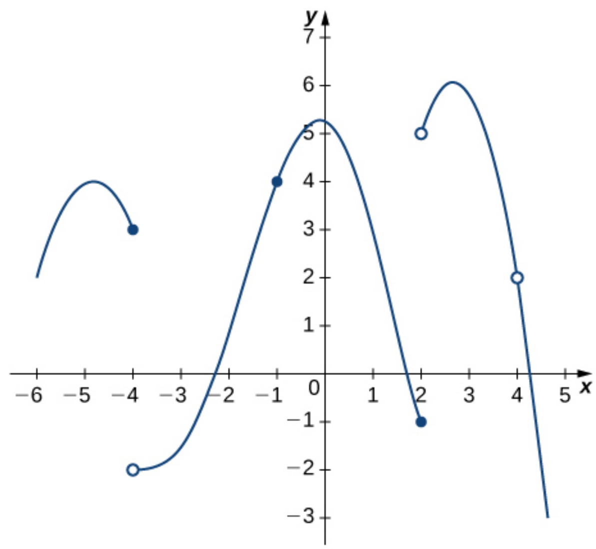
77. $f(x) = \begin{cases} 2x, & x \le 1 \\ \frac{2}{x}, & x > 1 \end{cases}$

For the following graphs,

a. determine for which values of $x = a$ the $\lim_{x \to a} f(x)$ exists but f is not continuous at $x = a$, and

b. determine for which values of $x = a$ the function is continuous but not differentiable at $x = a$.

78.



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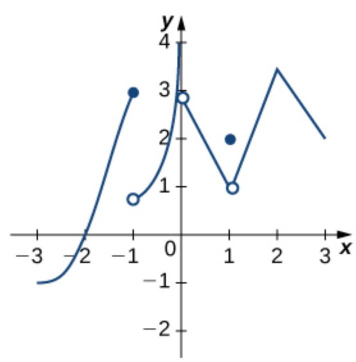
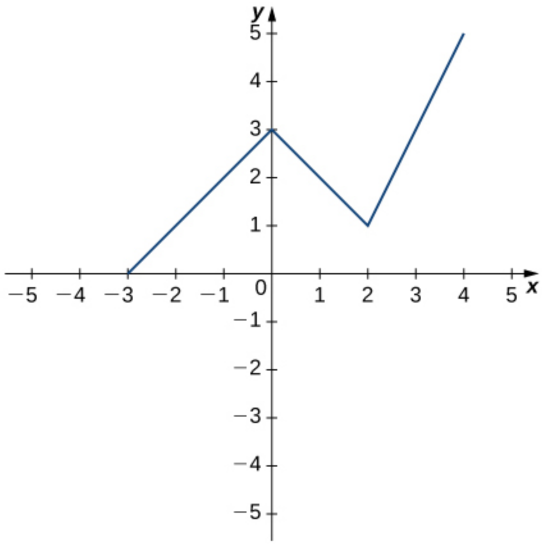


80. Use the graph to evaluate a. $f'(-0.5)$, b. $f'(0)$, c. $f'(1)$, d. $f'(2)$, and e. $f'(3)$, if it exists.



For the following functions, use $f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$ to find $f''(x)$.

81. $f(x) = 2 - 3x$

82. $f(x) = 4x^2$

83. $f(x) = x + \frac{1}{x}$

For the following exercises, use a calculator to graph $f(x)$. Determine the function $f'(x)$, then use a calculator to graph $f'(x)$.

84. **[T]** $f(x) = -\frac{5}{x}$

85. **[T]** $f(x) = 3x^2 + 2x + 4.$

86. **[T]** $f(x) = \sqrt{x} + 3x$

87. **[T]** $f(x) = \frac{1}{\sqrt{2x}}$

88. **[T]** $f(x) = 1 + x + \frac{1}{x}$

89. **[T]** $f(x) = x^3 + 1$

For the following exercises, describe what the two expressions represent in terms of each of the given situations. Be sure to include units.

a. $\frac{f(x+h) - f(x)}{h}$

b. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

90. $P(x)$ denotes the population of a city at time x in years.

91. $C(x)$ denotes the total amount of money (in thousands of dollars) spent on concessions by x customers at an amusement park.

92. $R(x)$ denotes the total cost (in thousands of dollars) of manufacturing x clock radios.

93. $g(x)$ denotes the grade (in percentage points) received on a test, given x hours of studying.

94. $B(x)$ denotes the cost (in dollars) of a sociology textbook at university bookstores in the United States in x years since 1990.

95. $p(x)$ denotes atmospheric pressure at an altitude of x feet.

96. Sketch the graph of a function $y = f(x)$ with all of the following properties:

a. $f'(x) > 0$ for $-2 \le x < 1$

b. $f'(2) = 0$

c. $f'(x) > 0$ for $x > 2$

d. $f(2) = 2$ and $f(0) = 1$

e. $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$

f. $f'(1)$ does not exist.

97. Suppose temperature T in degrees Fahrenheit at a height x in feet above the ground is given by $y = T(x)$.

a. Give a physical interpretation, with units, of $T'(x)$.

b. If we know that $T'(1000) = -0.1$, explain the physical meaning.

98. Suppose the total profit of a company is $y = P(x)$ thousand dollars when x units of an item are sold.

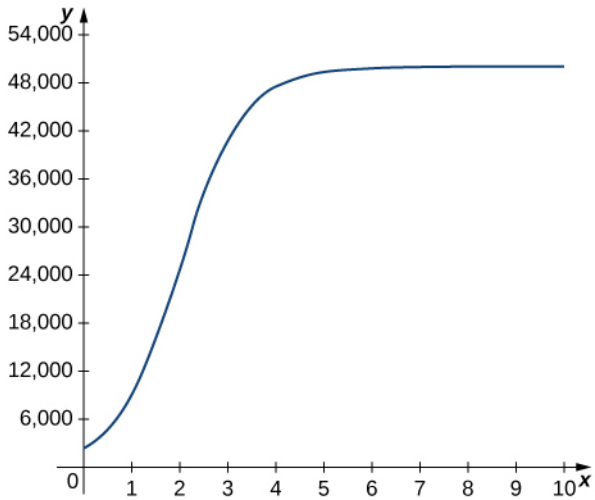
a. What does $\frac{P(b) - P(a)}{b - a}$ for $0 < a < b$ measure, and what are the units?

b. What does $P'(x)$ measure, and what are the units?

c. Suppose that $P'(30) = 5$, what is the approximate change in profit if the number of items sold increases from 30 to 31?

99. The graph in the following figure models the number of people $N(t)$ who have come down with the flu t weeks after its initial outbreak in a town with a population of $50,000$ citizens.

a. Describe what $N'(t)$ represents and how it behaves as t increases.
b. What does the derivative tell us about how this town is affected by the flu outbreak?



For the following exercises, use the following table, which shows the height h of the Saturn V rocket for the Apollo 11 mission t seconds after launch.

Time (seconds)	Height (meters)
0	0
1	2
2	4
3	13
4	25
5	32

100. What is the physical meaning of $h'(t)$? What are the units?

101. **[T]** Construct a table of values for $h'(t)$ and graph both $h(t)$ and $h'(t)$ on the same graph. (*Hint:* for **interior points**, estimate both the left limit and right limit and average them. An interior point of an interval I is an element of I which is not an endpoint of I.)

102. **[T]** The best linear fit to the data is given by $H(t) = 7.229t - 4.905,$ where H is the height of the rocket (in meters) and t is the time elapsed since takeoff. From this equation, determine $H'(t)$. Graph $H(t)$ with the given data and, on a separate coordinate plane, graph $H'(t)$.

103. **[T]** The best quadratic fit to the data is given by $G(t) = 1.429t^2 + 0.0857t - 0.1429,$ where G is the height of the rocket (in meters) and t is the time elapsed since takeoff. From this equation, determine $G'(t)$. Graph $G(t)$ with the given data and, on a separate coordinate plane, graph $G'(t)$.

104. **[T]** The best cubic fit to the data is given by $F(t) = 0.2037t^3 + 2.956t^2 - 2.705t + 0.4683,$ where F is the height of the rocket (in m) and t is the time elapsed since take off. From this equation, determine $F'(t)$. Graph $F(t)$ with the given data and, on a separate coordinate plane, graph $F'(t)$. Does the linear, quadratic, or cubic function fit the data best?

105. Using the best linear, quadratic, and cubic fits to the data, determine what $H''(t)$, $G''(t)$ and $F''(t)$ are. What are the physical meanings of $H''(t)$, $G''(t)$ and $F''(t)$, and what are their units?