2.1 EXERCISES

For the following exercises, points $P(1, 2)$ and $Q(x, y)$ are on the graph of the function $f(x) = x^2 + 1$.

1. **[T]** Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(x, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

x	y	$Q(x, y)$	m_{sec}
1.1	a.	e.	i.
1.01	b.	f.	j.
1.001	c.	g.	k.
1.0001	d.	h.	l.

2. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the line tangent to f at $x = 1$.

3. Use the value in the preceding exercise to find the equation of the tangent line at point P. Graph $f(x)$ and the tangent line.

For the following exercises, points $P(1, 1)$ and $Q(x, y)$ are on the graph of the function $f(x) = x^3$.

4. **[T]** Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(x, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

x	y	$Q(x, y)$	m_{sec}
1.1	a.	e.	i.
1.01	b.	f.	j.
1.001	c.	g.	k.
1.0001	d.	h.	l.

5. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to f at $x = 1$.

6. Use the value in the preceding exercise to find the equation of the tangent line at point P. Graph $f(x)$ and the tangent line.

For the following exercises, points $P(4, 2)$ and $Q(x, y)$ are on the graph of the function $f(x) = \sqrt{x}$.

7. **[T]** Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(x, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

x	y	$Q(x, y)$	m_{sec}
4.1	a.	e.	i.
4.01	b.	f.	j.
4.001	c.	g.	k.
4.0001	d.	h.	l.

8. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to f at $x = 4$.

9. Use the value in the preceding exercise to find the equation of the tangent line at point P.

For the following exercises, points $P(1.5, 0)$ and $Q(\phi, y)$ are on the graph of the function $f(\phi) = \cos(\pi\phi)$.

10. **[T]** Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(\varphi, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

x	y	$Q(\phi, y)$	m_{sec}
1.4	a.	e.	i.
1.49	b.	f.	j.
1.499	c.	g.	k.
1.4999	d.	h.	l.

11. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the tangent line to f at $\varphi = 1.5$.

12. Use the value in the preceding exercise to find the equation of the tangent line at point P.

For the following exercises, points $P(-1, -1)$ and $Q(x, y)$ are on the graph of the function $f(x) = \frac{1}{x}$.

13. **[T]** Complete the following table with the appropriate values: y-coordinate of Q, the point $Q(x, y)$, and the slope of the secant line passing through points P and Q. Round your answer to eight significant digits.

x	y	$Q(x, y)$	m_{sec}
−1.05	a.	e.	i.
−1.01	b.	f.	j.
−1.005	c.	g.	k.
−1.001	d.	h.	l.

14. Use the values in the right column of the table in the preceding exercise to guess the value of the slope of the line tangent to f at $x = -1$.

15. Use the value in the preceding exercise to find the equation of the tangent line at point P.

For the following exercises, the position function of a ball dropped from the top of a 200-meter tall building is given by $s(t) = 200 - 4.9t^2$, where position s is measured in meters and time t is measured in seconds. Round your answer to eight significant digits.

16. **[T]** Compute the average velocity of the ball over the given time intervals.
   a. [4.99, 5]
   b. [5, 5.01]
   c. [4.999, 5]
   d. [5, 5.001]

17. Use the preceding exercise to guess the instantaneous velocity of the ball at $t = 5$ sec.

For the following exercises, consider a stone tossed into the air from ground level with an initial velocity of 15 m/sec. Its height in meters at time t seconds is $h(t) = 15t - 4.9t^2$.

18. **[T]** Compute the average velocity of the stone over the given time intervals.
   a. [1, 1.05]
   b. [1, 1.01]
   c. [1, 1.005]
   d. [1, 1.001]

19. Use the preceding exercise to guess the instantaneous velocity of the stone at $t = 1$ sec.

For the following exercises, consider a rocket shot into the air that then returns to Earth. The height of the rocket in meters is given by $h(t) = 600 + 78.4t - 4.9t^2$, where t is measured in seconds.

20. **[T]** Compute the average velocity of the rocket over the given time intervals.
   a. [9, 9.01]
   b. [8.99, 9]
   c. [9, 9.001]
   d. [8.999, 9]

21. Use the preceding exercise to guess the instantaneous velocity of the rocket at $t = 9$ sec.

For the following exercises, consider an athlete running a 40-m dash. The position of the athlete is given by $d(t) = \frac{t^3}{6} + 4t$, where d is the position in meters and t is the time elapsed, measured in seconds.

22. **[T]** Compute the average velocity of the runner over the given time intervals.

a. $[1.95, 2.05]$

b. $[1.995, 2.005]$

c. $[1.9995, 2.0005]$

d. $[2, 2.00001]$

23. Use the preceding exercise to guess the instantaneous velocity of the runner at $t = 2$ sec.

For the following exercises, consider the function $f(x) = |x|$.

24. Sketch the graph of f over the interval $[-1, 2]$ and shade the region above the x-axis.

25. Use the preceding exercise to find the aproximate value of the area between the x-axis and the graph of f over the interval $[-1, 2]$ using rectangles. For the rectangles, use the square units, and approximate both above and below the lines. Use geometry to find the exact answer.

For the following exercises, consider the function $f(x) = \sqrt{1 - x^2}$. (*Hint*: This is the upper half of a circle of radius 1 positioned at $(0, 0)$.)

26. Sketch the graph of f over the interval $[-1, 1]$.

27. Use the preceding exercise to find the aproximate area between the x-axis and the graph of f over the interval $[-1, 1]$ using rectangles. For the rectangles, use squares 0.4 by 0.4 units, and approximate both above and below the lines. Use geometry to find the exact answer.

For the following exercises, consider the function $f(x) = -x^2 + 1$.

28. Sketch the graph of f over the interval $[-1, 1]$.

29. Approximate the area of the region between the x-axis and the graph of f over the interval $[-1, 1]$.