2.2 EXERCISES

For the following exercises, consider the function $f(x) = \frac{x^2 - 1}{|x - 1|}$.

30. **[T]** Complete the following table for the function. Round your solutions to four decimal places.

x	$f(x)$		x	$f(x)$
0.9	a.		1.1	e.
0.99	b.		1.01	f.
0.999	c.		1.001	g.
0.9999	d.		1.0001	h.

31. What do your results in the preceding exercise indicate about the two-sided limit $\lim_{x \to 1} f(x)$? Explain your response.

For the following exercises, consider the function $f(x) = (1 + x)^{1/x}$.

32. **[T]** Make a table showing the values of f for $x = -0.01, -0.001, -0.0001, -0.00001$ and for $x = 0.01, 0.001, 0.0001, 0.00001$. Round your solutions to five decimal places.

x	$f(x)$		x	$f(x)$
−0.01	a.		0.01	e.
−0.001	b.		0.001	f.
−0.0001	c.		0.0001	g.
−0.00001	d.		0.00001	h.

33. What does the table of values in the preceding exercise indicate about the function $f(x) = (1 + x)^{1/x}$?

34. To which mathematical constant does the limit in the preceding exercise appear to be getting closer?

In the following exercises, use the given values to set up a table to evaluate the limits. Round your solutions to eight decimal places.

35. **[T]** $\lim_{x \to 0} \frac{\sin 2x}{x}$; $\pm 0.1, \pm 0.01, \pm 0.001, \pm .0001$

x	$\frac{\sin 2x}{x}$		x	$\frac{\sin 2x}{x}$
−0.1	a.		0.1	e.
−0.01	b.		0.01	f.
−0.001	c.		0.001	g.
−0.0001	d.		0.0001	h.

36. **[T]** $\lim_{x \to 0} \frac{\sin 3x}{x}$ ±0.1, ±0.01, ±0.001, ±0.0001

X	$\frac{\sin 3x}{x}$		x	$\frac{\sin 3x}{x}$
−0.1	a.		0.1	e.
−0.01	b.		0.01	f.
−0.001	c.		0.001	g.
−0.0001	d.		0.0001	h.

37. Use the preceding two exercises to conjecture (guess) the value of the following limit: $\lim_{x \to 0} \frac{\sin ax}{x}$ for a, a positive real value.

[T] In the following exercises, set up a table of values to find the indicated limit. Round to eight digits.

38. $\lim\limits_{x \to 2} \frac{x^2 - 4}{x^2 + x - 6}$

x	$\frac{x^2-4}{x^2+x-6}$		x	$\frac{x^2-4}{x^2+x-6}$
1.9	a.		2.1	e.
1.99	b.		2.01	f.
1.999	c.		2.001	g.
1.9999	d.		2.0001	h.

39. $\lim\limits_{x \to 1} (1 - 2x)$

x	$1-2x$		x	$1-2x$
0.9	a.		1.1	e.
0.99	b.		1.01	f.
0.999	c.		1.001	g.
0.9999	d.		1.0001	h.

40. $\lim\limits_{x \to 0} \frac{5}{1 - e^{1/x}}$

x	$\frac{5}{1-e^{1/x}}$		x	$\frac{5}{1-e^{1/x}}$
−0.1	a.		0.1	e.
−0.01	b.		0.01	f.
−0.001	c.		0.001	g.
−0.0001	d.		0.0001	h.

41. $\lim\limits_{z \to 0} \frac{z - 1}{z^2 (z + 3)}$

z	$\frac{z-1}{z^2(z+3)}$		z	$\frac{z-1}{z^2(z+3)}$
−0.1	a.		0.1	e.
−0.01	b.		0.01	f.
−0.001	c.		0.001	g.
−0.0001	d.		0.0001	h.

42. $\lim\limits_{t \to 0^+} \frac{\cos t}{t}$

t	$\frac{\cos t}{t}$
0.1	a.
0.01	b.
0.001	c.
0.0001	d.

43. $\lim\limits_{x \to 2} \frac{1 - \frac{2}{x}}{x^2 - 4}$

x	$\frac{1-\frac{2}{x}}{x^2-4}$		x	$\frac{1-\frac{2}{x}}{x^2-4}$
1.9	a.		2.1	e.
1.99	b.		2.01	f.
1.999	c.		2.001	g.
1.9999	d.		2.0001	h.

[T] In the following exercises, set up a table of values and round to eight significant digits. Based on the table of values, make a guess about what the limit is. Then, use a calculator to graph the function and determine the limit. Was the conjecture correct? If not, why does the method of tables fail?

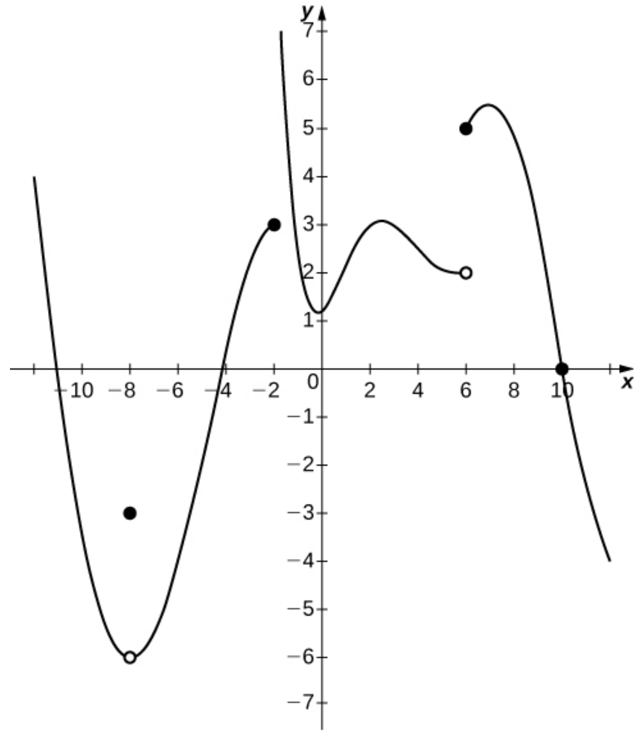
44. $\lim_{\theta \to 0} \sin\left(\frac{\pi}{\theta}\right)$

$\boldsymbol{\theta}$	$\mathbf{sin}\left(\frac{\boldsymbol{\pi}}{\boldsymbol{\theta}}\right)$		$\boldsymbol{\theta}$	$\mathbf{sin}\left(\frac{\boldsymbol{\pi}}{\boldsymbol{\theta}}\right)$
−0.1	a.		0.1	e.
−0.01	b.		0.01	f.
−0.001	c.		0.001	g.
−0.0001	d.		0.0001	h.

45. $\lim_{\alpha \to 0^+} \frac{1}{\alpha}\cos\left(\frac{\pi}{\alpha}\right)$

a	$\frac{\mathbf{1}}{\boldsymbol{\alpha}}\mathbf{cos}\left(\frac{\boldsymbol{\pi}}{\boldsymbol{\alpha}}\right)$
0.1	a.
0.01	b.
0.001	c.
0.0001	d.

In the following exercises, consider the graph of the function $y = f(x)$ shown here. Which of the statements about $y = f(x)$ are true and which are false? Explain why a statement is false.



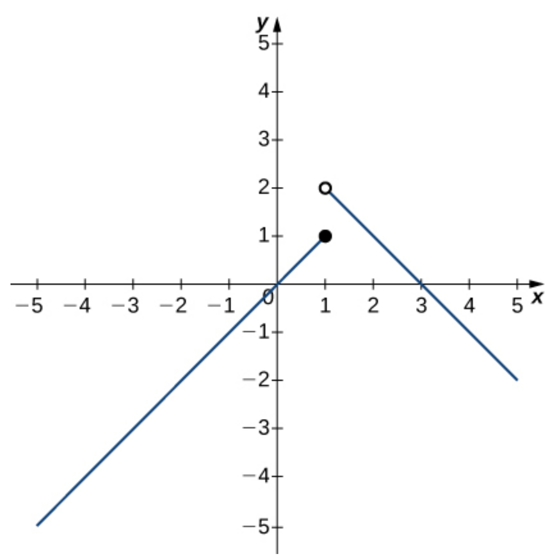
46. $\lim_{x \to 10} f(x) = 0$

47. $\lim_{x \to -2^+} f(x) = 3$

48. $\lim_{x \to -8} f(x) = f(-8)$

49. $\lim_{x \to 6} f(x) = 5$

In the following exercises, use the following graph of the function $y = f(x)$ to find the values, if possible. Estimate when necessary.



50. $\lim\limits_{x \to 1^-} f(x)$

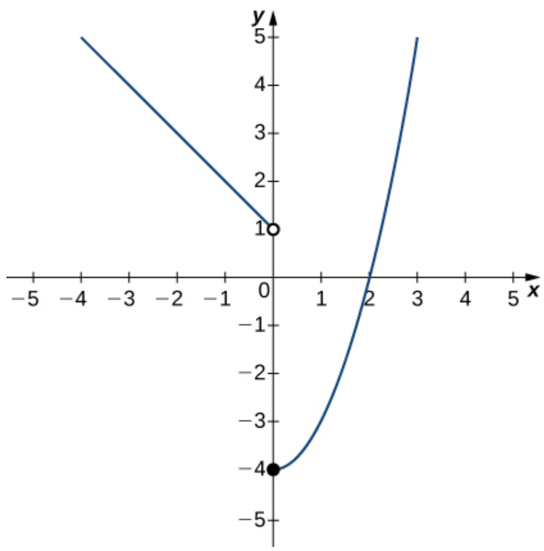
51. $\lim\limits_{x \to 1^+} f(x)$

52. $\lim\limits_{x \to 1} f(x)$

53. $\lim\limits_{x \to 2} f(x)$

54. $f(1)$

In the following exercises, use the graph of the function $y = f(x)$ shown here to find the values, if possible. Estimate when necessary.



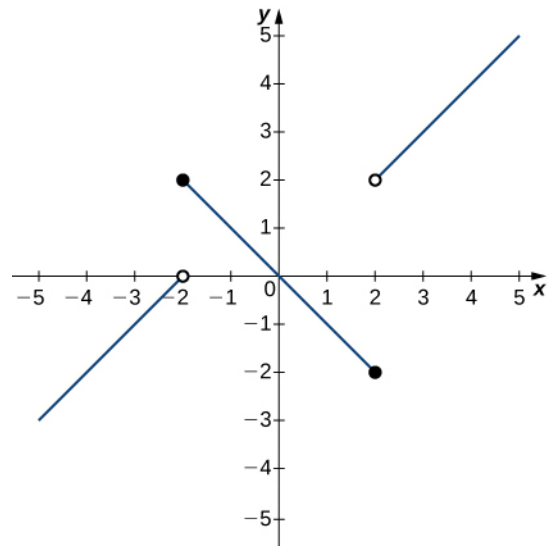
55. $\lim\limits_{x \to 0^-} f(x)$

56. $\lim\limits_{x \to 0^+} f(x)$

57. $\lim\limits_{x \to 0} f(x)$

58. $\lim\limits_{x \to 2} f(x)$

In the following exercises, use the graph of the function $y = f(x)$ shown here to find the values, if possible. Estimate when necessary.



59. $\lim\limits_{x \to -2^-} f(x)$

60. $\lim\limits_{x \to -2^+} f(x)$

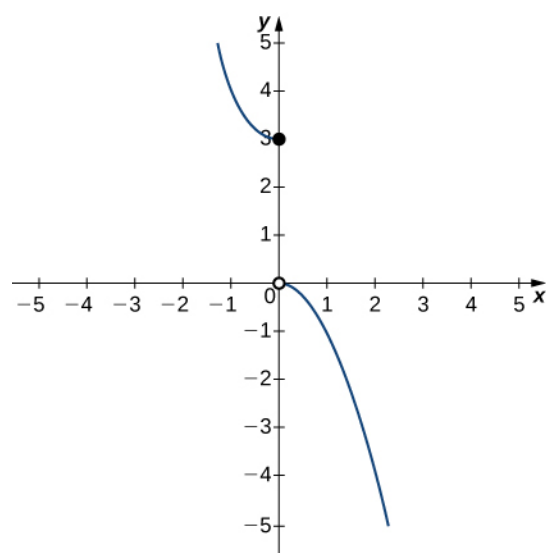
61. $\lim\limits_{x \to -2} f(x)$

62. $\lim\limits_{x \to 2^-} f(x)$

63. $\lim\limits_{x \to 2^+} f(x)$

64. $\lim\limits_{x \to 2} f(x)$

In the following exercises, use the graph of the function $y = g(x)$ shown here to find the values, if possible. Estimate when necessary.

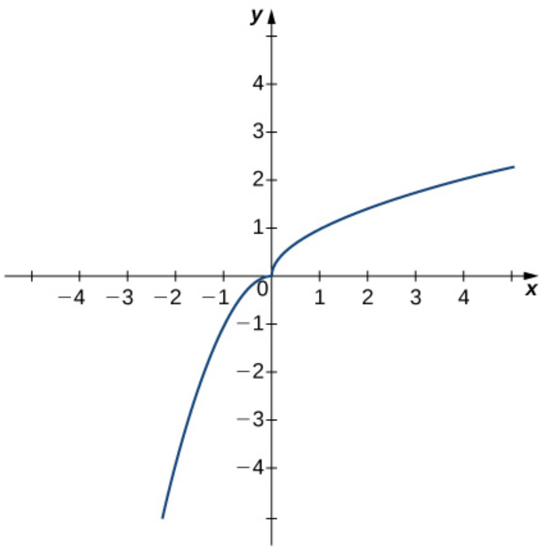


65. $\lim\limits_{x \to 0^-} g(x)$

66. $\lim_{x \to 0^+} g(x)$

67. $\lim_{x \to 0} g(x)$

In the following exercises, use the graph of the function $y = h(x)$ shown here to find the values, if possible. Estimate when necessary.

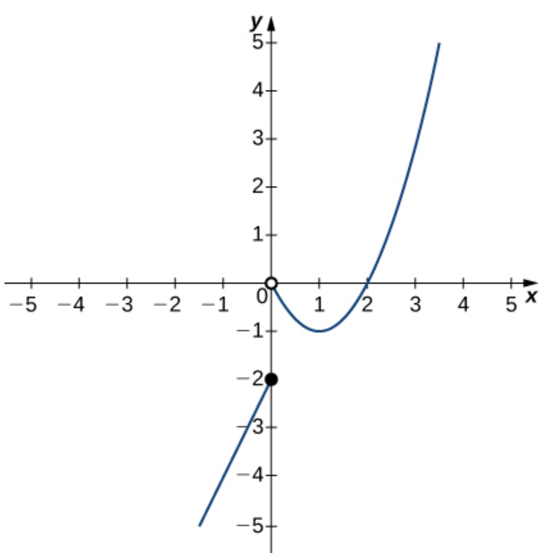


68. $\lim_{x \to 0^-} h(x)$

69. $\lim_{x \to 0^+} h(x)$

70. $\lim_{x \to 0} h(x)$

In the following exercises, use the graph of the function $y = f(x)$ shown here to find the values, if possible. Estimate when necessary.



71. $\lim_{x \to 0^-} f(x)$

72. $\lim_{x \to 0^+} f(x)$

73. $\lim_{x \to 0} f(x)$

74. $\lim_{x \to 1} f(x)$

75. $\lim_{x \to 2} f(x)$

In the following exercises, sketch the graph of a function with the given properties.

76. $\lim_{x \to 2} f(x) = 1$, $\lim_{x \to 4^-} f(x) = 3$, $\lim_{x \to 4^+} f(x) = 6$, $f(4)$ is not defined.

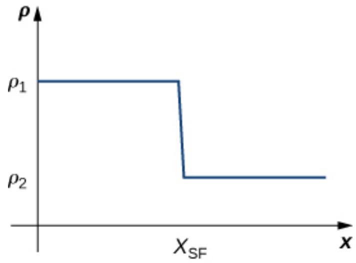
77. $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to -1^-} f(x) = -\infty$, $\lim_{x \to -1^+} f(x) = \infty$, $\lim_{x \to 0} f(x) = f(0)$, $f(0) = 1$, $\lim_{x \to \infty} f(x) = -\infty$

78. $\lim_{x \to -\infty} f(x) = 2$, $\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 2$, $f(0) = \frac{-1}{3}$

79. $\lim_{x \to -\infty} f(x) = 2$, $\lim_{x \to -2} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = 2$, $f(0) = 0$

80. $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $f(0) = -1$, $\lim_{x \to 1^-} f(x) = -\infty$, $\lim_{x \to 1^+} f(x) = \infty$, $\lim_{x \to \infty} f(x) = 0$

81. Shock waves arise in many physical applications, ranging from supernovas to detonation waves. A graph of the density of a shock wave with respect to distance, x, is shown here. We are mainly interested in the location of the front of the shock, labeled x_{SF} in the diagram.



a. Evaluate $\lim_{x \to x_{SF}^{+}} \rho(x)$.

b. Evaluate $\lim_{x \to x_{SF}^{-}} \rho(x)$.

c. Evaluate $\lim_{x \to x_{SF}} \rho(x)$. Explain the physical meanings behind your answers.

82. A track coach uses a camera with a fast shutter to estimate the position of a runner with respect to time. A table of the values of position of the athlete versus time is given here, where x is the position in meters of the runner and t is time in seconds. What is $\lim_{t \to 2} x(t)$? What does it mean physically?

t **(sec)**	***x*** **(m)**
1.75	4.5
1.95	6.1
1.99	6.42
2.01	6.58
2.05	6.9
2.25	8.5