2.5 EXERCISES

In the following exercises, write the appropriate $\varepsilon - \delta$ definition for each of the given statements.

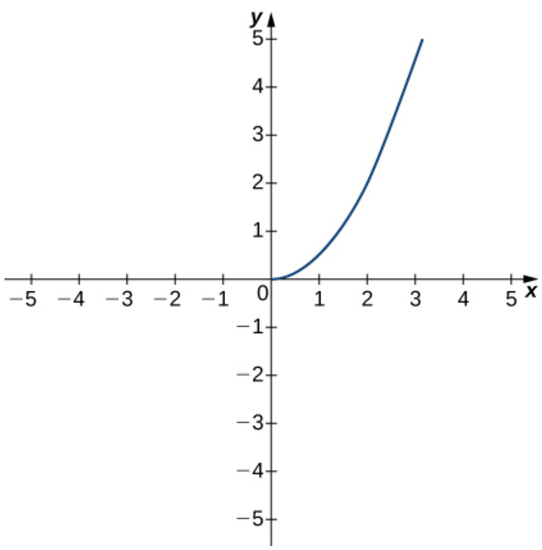
176. $\lim\limits_{x \to a} f(x) = N$

177. $\lim\limits_{t \to b} g(t) = M$

178. $\lim\limits_{x \to c} h(x) = L$

179. $\lim\limits_{x \to a} \varphi(x) = A$

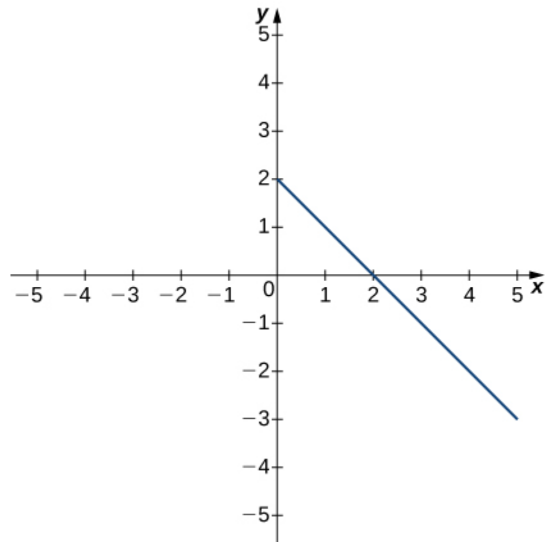
The following graph of the function f satisfies $\lim\limits_{x \to 2} f(x) = 2$. In the following exercises, determine a value of $\delta > 0$ that satisfies each statement.



180. If $0 < |x - 2| < \delta$, then $|f(x) - 2| < 1$.

181. If $0 < |x - 2| < \delta$, then $|f(x) - 2| < 0.5$.

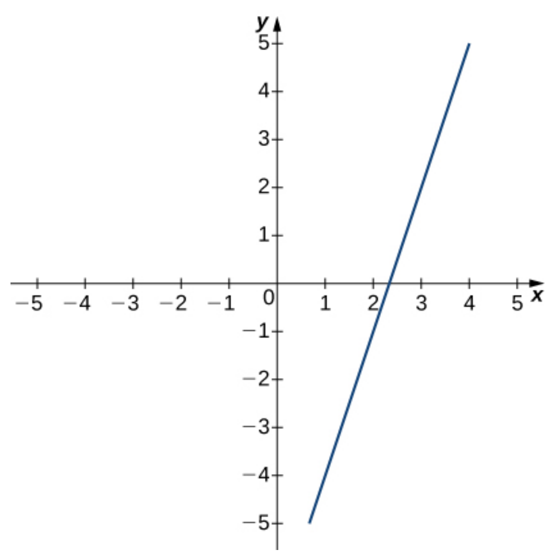
The following graph of the function f satisfies $\lim\limits_{x \to 3} f(x) = -1$. In the following exercises, determine a value of $\delta > 0$ that satisfies each statement.



182. If $0 < |x - 3| < \delta$, then $|f(x) + 1| < 1$.

183. If $0 < |x - 3| < \delta$, then $|f(x) + 1| < 2$.

The following graph of the function f satisfies $\lim\limits_{x \to 3} f(x) = 2$. In the following exercises, for each value of ε, find a value of $\delta > 0$ such that the precise definition of limit holds true.



184. $\varepsilon = 1.5$

185. $\varepsilon = 3$

[T] In the following exercises, use a graphing calculator to find a number δ such that the statements hold true.

186. $\left|\sin(2x) - \frac{1}{2}\right| < 0.1$, whenever $\left|x - \frac{\pi}{12}\right| < \delta$

187. $\left|\sqrt{x-4}-2\right| < 0.1,$ whenever $|x-8| < \delta$

In the following exercises, use the precise definition of limit to prove the given limits.

188. $\lim\limits_{x \to 2}(5x+8) = 18$

189. $\lim\limits_{x \to 3}\dfrac{x^2-9}{x-3} = 6$

190. $\lim\limits_{x \to 2}\dfrac{2x^2-3x-2}{x-2} = 5$

191. $\lim\limits_{x \to 0}x^4 = 0$

192. $\lim\limits_{x \to 2}(x^2+2x) = 8$

In the following exercises, use the precise definition of limit to prove the given one-sided limits.

193. $\lim\limits_{x \to 5^-}\sqrt{5-x} = 0$

194.
$\lim\limits_{x \to 0^+} f(x) = -2,$ where $f(x) = \begin{cases} 8x-3, & \text{if } x < 0 \\ 4x-2, & \text{if } x \geq 0 \end{cases}.$

195. $\lim\limits_{x \to 1^-} f(x) = 3,$ where $f(x) = \begin{cases} 5x-2, & \text{if } x < 1 \\ 7x-1, & \text{if } x \geq 1 \end{cases}.$

In the following exercises, use the precise definition of limit to prove the given infinite limits.

196. $\lim\limits_{x \to 0}\dfrac{1}{x^2} = \infty$

197. $\lim\limits_{x \to -1}\dfrac{3}{(x+1)^2} = \infty$

198. $\lim\limits_{x \to 2} -\dfrac{1}{(x-2)^2} = -\infty$

199. An engineer is using a machine to cut a flat square of Aerogel of area 144 cm^2. If there is a maximum error tolerance in the area of 8 cm^2, how accurately must the engineer cut on the side, assuming all sides have the same length? How do these numbers relate to $\delta,$ ε, a, and L?

200. Use the precise definition of limit to prove that the following limit does not exist: $\lim\limits_{x \to 1}\dfrac{|x-1|}{x-1}.$

201. Using precise definitions of limits, prove that $\lim\limits_{x \to 0} f(x)$ does not exist, given that $f(x)$ is the ceiling function. (*Hint*: Try any $\delta < 1.$)

202. Using precise definitions of limits, prove that $\lim\limits_{x \to 0} f(x)$ does not exist: $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}.$ (*Hint*: Think about how you can always choose a rational number $0 < r < d,$ but $|f(r)-0| = 1.$)

203. Using precise definitions of limits, determine $\lim\limits_{x \to 0} f(x)$ for $f(x) = \begin{cases} x \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}.$ (*Hint*: Break into two cases, x rational and x irrational.)

204. Using the function from the previous exercise, use the precise definition of limits to show that $\lim\limits_{x \to a} f(x)$ does not exist for $a \neq 0.$

For the following exercises, suppose that $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} g(x) = M$ both exist. Use the precise definition of limits to prove the following limit laws:

205. $\lim\limits_{x \to a}(f(x)+g(x)) = L+M$

206. $\lim\limits_{x \to a}[cf(x)] = cL$ for any real constant c (*Hint*: Consider two cases: $c = 0$ and $c \neq 0.$)

207. $\lim\limits_{x \to a}[f(x)g(x)] = LM.$ (*Hint*: $|f(x)g(x)-LM| =$ $|f(x)g(x)-f(x)M+f(x)M-LM| \leq |f(x)||g(x)-M|+|M||f(x)-L|.$)