2.3 EXERCISES

In the following exercises, use the limit laws to evaluate each limit. Justify each step by indicating the appropriate limit law(s).

83. $\lim\limits_{x \to 0} \left(4x^2 - 2x + 3\right)$

84. $\lim\limits_{x \to 1} \dfrac{x^3 + 3x^2 + 5}{4 - 7x}$

85. $\lim\limits_{x \to -2} \sqrt{x^2 - 6x + 3}$

86. $\lim\limits_{x \to -1} (9x + 1)^2$

In the following exercises, use direct substitution to evaluate each limit.

87. $\lim\limits_{x \to 7} x^2$

88. $\lim\limits_{x \to -2} \left(4x^2 - 1\right)$

89. $\lim\limits_{x \to 0} \dfrac{1}{1 + \sin x}$

90. $\lim\limits_{x \to 2} e^{2x - x^2}$

91. $\lim\limits_{x \to 1} \dfrac{2 - 7x}{x + 6}$

92. $\lim\limits_{x \to 3} \ln e^{3x}$

In the following exercises, use direct substitution to show that each limit leads to the indeterminate form $0/0$. Then, evaluate the limit.

93. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$

94. $\lim\limits_{x \to 2} \dfrac{x - 2}{x^2 - 2x}$

95. $\lim\limits_{x \to 6} \dfrac{3x - 18}{2x - 12}$

96. $\lim\limits_{h \to 0} \dfrac{(1 + h)^2 - 1}{h}$

97. $\lim\limits_{t \to 9} \dfrac{t - 9}{\sqrt{t} - 3}$

98. $\lim\limits_{h \to 0} \dfrac{\frac{1}{a+h} - \frac{1}{a}}{h}$, where a is a non-zero real-valued constant

99. $\lim\limits_{\theta \to \pi} \dfrac{\sin\theta}{\tan\theta}$

100. $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$

101. $\lim\limits_{x \to 1/2} \dfrac{2x^2 + 3x - 2}{2x - 1}$

102. $\lim\limits_{x \to -3} \dfrac{\sqrt{x + 4} - 1}{x + 3}$

In the following exercises, use direct substitution to obtain an undefined expression. Then, use the method of **Example 2.23** to simplify the function to help determine the limit.

103. $\lim\limits_{x \to -2^-} \dfrac{2x^2 + 7x - 4}{x^2 + x - 2}$

104. $\lim\limits_{x \to -2^+} \dfrac{2x^2 + 7x - 4}{x^2 + x - 2}$

105. $\lim\limits_{x \to 1^-} \dfrac{2x^2 + 7x - 4}{x^2 + x - 2}$

106. $\lim\limits_{x \to 1^+} \dfrac{2x^2 + 7x - 4}{x^2 + x - 2}$

In the following exercises, assume that $\lim\limits_{x \to 6} f(x) = 4$, $\lim\limits_{x \to 6} g(x) = 9$, and $\lim\limits_{x \to 6} h(x) = 6$. Use these three facts and the limit laws to evaluate each limit.

107. $\lim\limits_{x \to 6} 2f(x)g(x)$

108. $\lim\limits_{x \to 6} \dfrac{g(x) - 1}{f(x)}$

109. $\lim\limits_{x \to 6} \left(f(x) + \frac{1}{3}g(x)\right)$

110. $\lim\limits_{x \to 6} \dfrac{(h(x))^3}{2}$

111. $\lim\limits_{x \to 6} \sqrt{g(x) - f(x)}$

112. $\lim\limits_{x \to 6} x \cdot h(x)$

113. $\lim_{x \to 6} [(x+1) \cdot f(x)]$

114. $\lim_{x \to 6} (f(x) \cdot g(x) - h(x))$

[T] In the following exercises, use a calculator to draw the graph of each piecewise-defined function and study the graph to evaluate the given limits.

115. $f(x) = \begin{cases} x^2, & x \le 3 \\ x+4, & x > 3 \end{cases}$

a. $\lim_{x \to 3^-} f(x)$

b. $\lim_{x \to 3^+} f(x)$

116. $g(x) = \begin{cases} x^3 - 1, & x \le 0 \\ 1, & x > 0 \end{cases}$

a. $\lim_{x \to 0^-} g(x)$

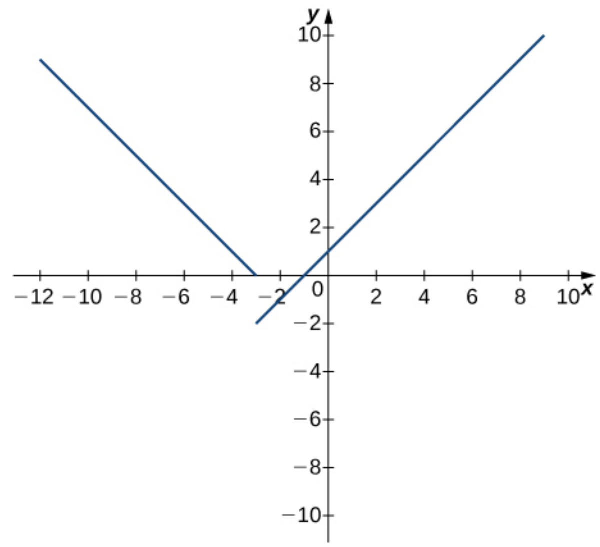
b. $\lim_{x \to 0^+} g(x)$

117. $h(x) = \begin{cases} x^2 - 2x + 1, & x < 2 \\ 3 - x, & x \ge 2 \end{cases}$

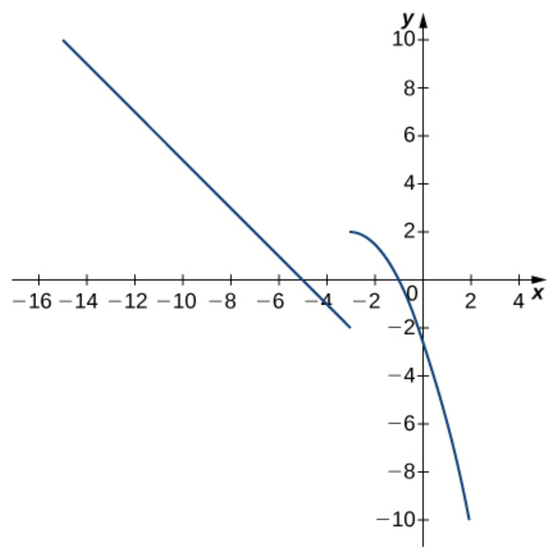
a. $\lim_{x \to 2^-} h(x)$

b. $\lim_{x \to 2^+} h(x)$

In the following exercises, use the following graphs and the limit laws to evaluate each limit.



$y = f(x)$



$y = g(x)$

118. $\lim_{x \to -3^+} (f(x) + g(x))$

119. $\lim_{x \to -3^-} (f(x) - 3g(x))$

120. $\lim_{x \to 0} \frac{f(x)g(x)}{3}$

121. $\lim_{x \to -5} \frac{2 + g(x)}{f(x)}$

122. $\lim_{x \to 1} (f(x))^2$

123. $\lim_{x \to 1} \sqrt[3]{f(x) - g(x)}$

124. $\lim_{x \to -7} (x \cdot g(x))$

125. $\lim_{x \to -9} [x \cdot f(x) + 2 \cdot g(x)]$

126. **[T]** True or False? If $2x - 1 \le g(x) \le x^2 - 2x + 3$, then $\lim_{x \to 2} g(x) = 0$.

For the following problems, evaluate the limit using the squeeze theorem. Use a calculator to graph the functions $f(x)$, $g(x)$, and $h(x)$ when possible.

127. **[T]** $\lim_{\theta \to 0} \theta^2 \cos\left(\frac{1}{\theta}\right)$

128. $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} 0, & x \text{ rational} \\ x^2, & x \text{ irrrational} \end{cases}$

129. **[T]** In physics, the magnitude of an electric field generated by a point charge at a distance r in vacuum is governed by Coulomb's law: $E(r) = \frac{q}{4\pi\varepsilon_0 r^2}$, where E represents the magnitude of the electric field, q is the charge of the particle, r is the distance between the particle and where the strength of the field is measured, and $\frac{1}{4\pi\varepsilon_0}$ is Coulomb's constant: $8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2$.

a. Use a graphing calculator to graph $E(r)$ given that the charge of the particle is $q = 10^{-10}$.
b. Evaluate $\lim_{r \to 0^+} E(r)$. What is the physical meaning of this quantity? Is it physically relevant? Why are you evaluating from the right?

130. **[T]** The density of an object is given by its mass divided by its volume: $\rho = m/V$.

a. Use a calculator to plot the volume as a function of density $(V = m/\rho)$, assuming you are examining something of mass 8 kg ($m = 8$).
b. Evaluate $\lim_{\rho \to 0^+} V(\rho)$ and explain the physical meaning.